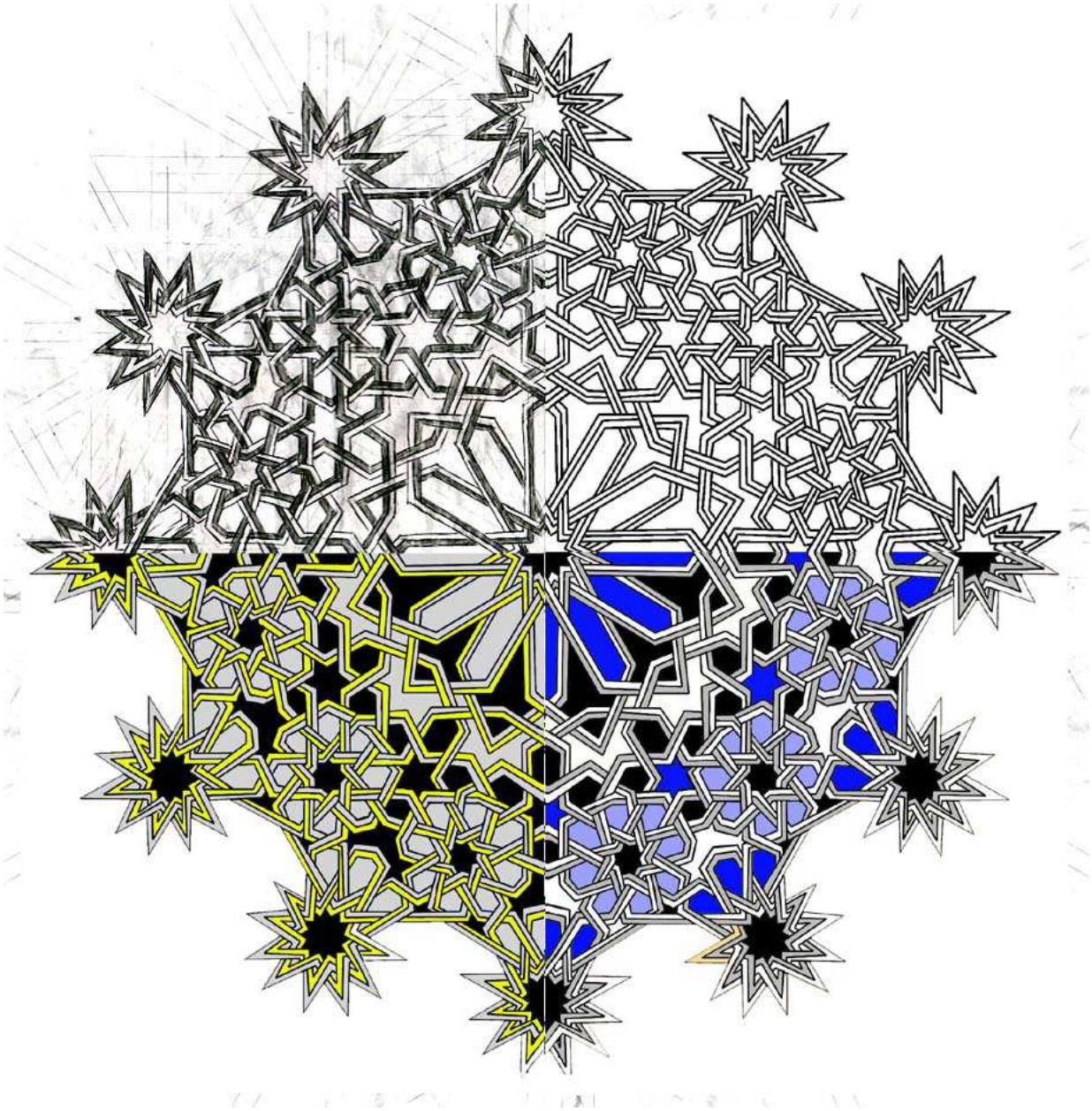
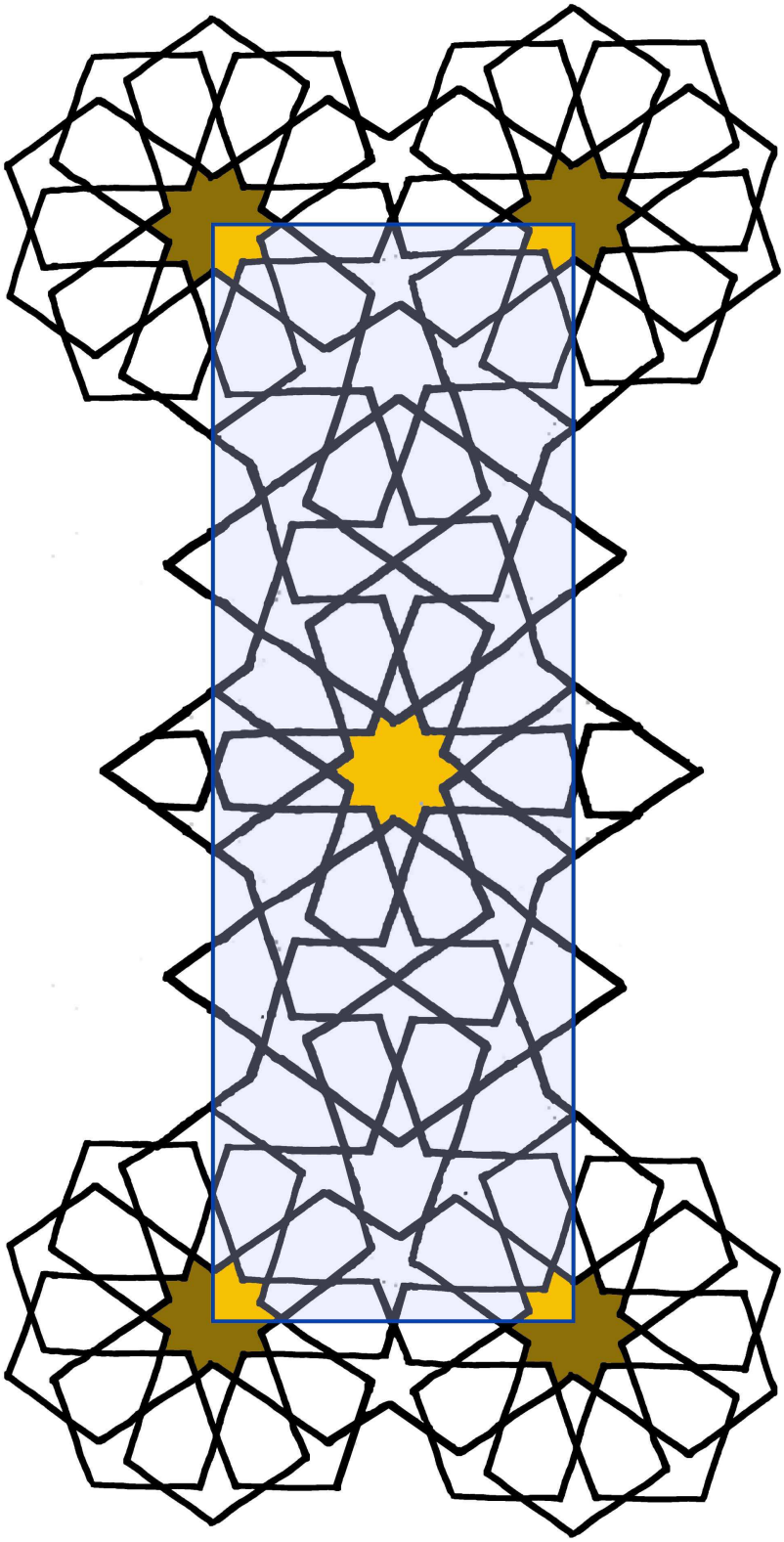
La géométrie des Arabesques



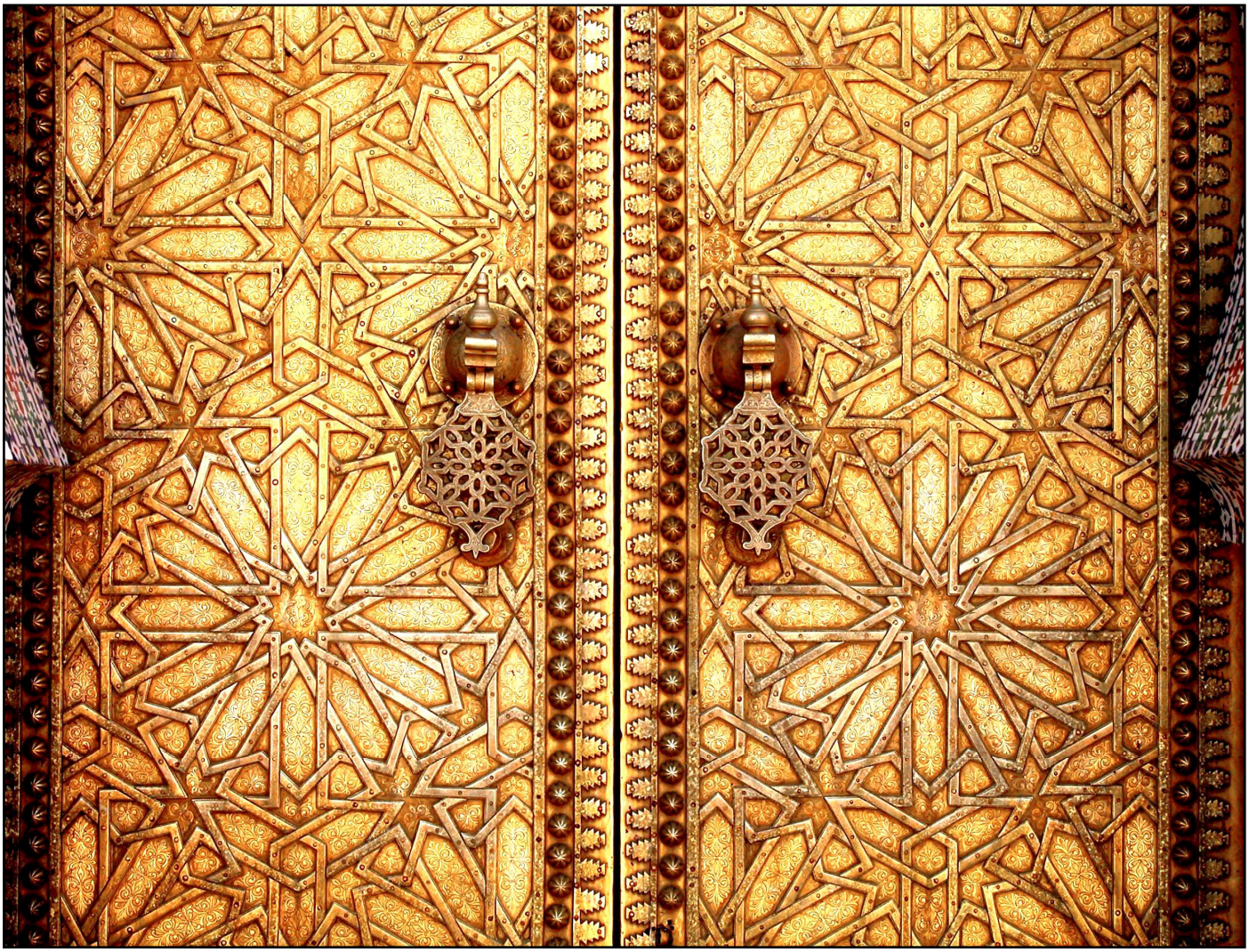
VIII. DESSIN :
LE SYSTEME HEXAGONAL
LE SYTEME PENTAGONAL

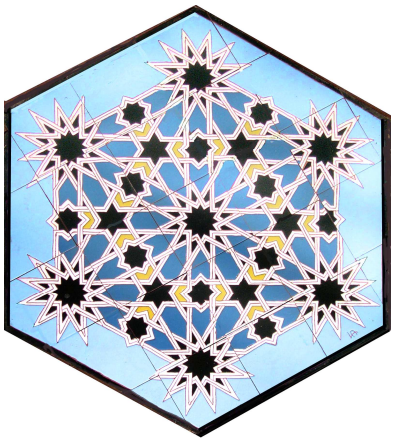
Composition d’étoiles à dix formant un système convergent pentagonal.



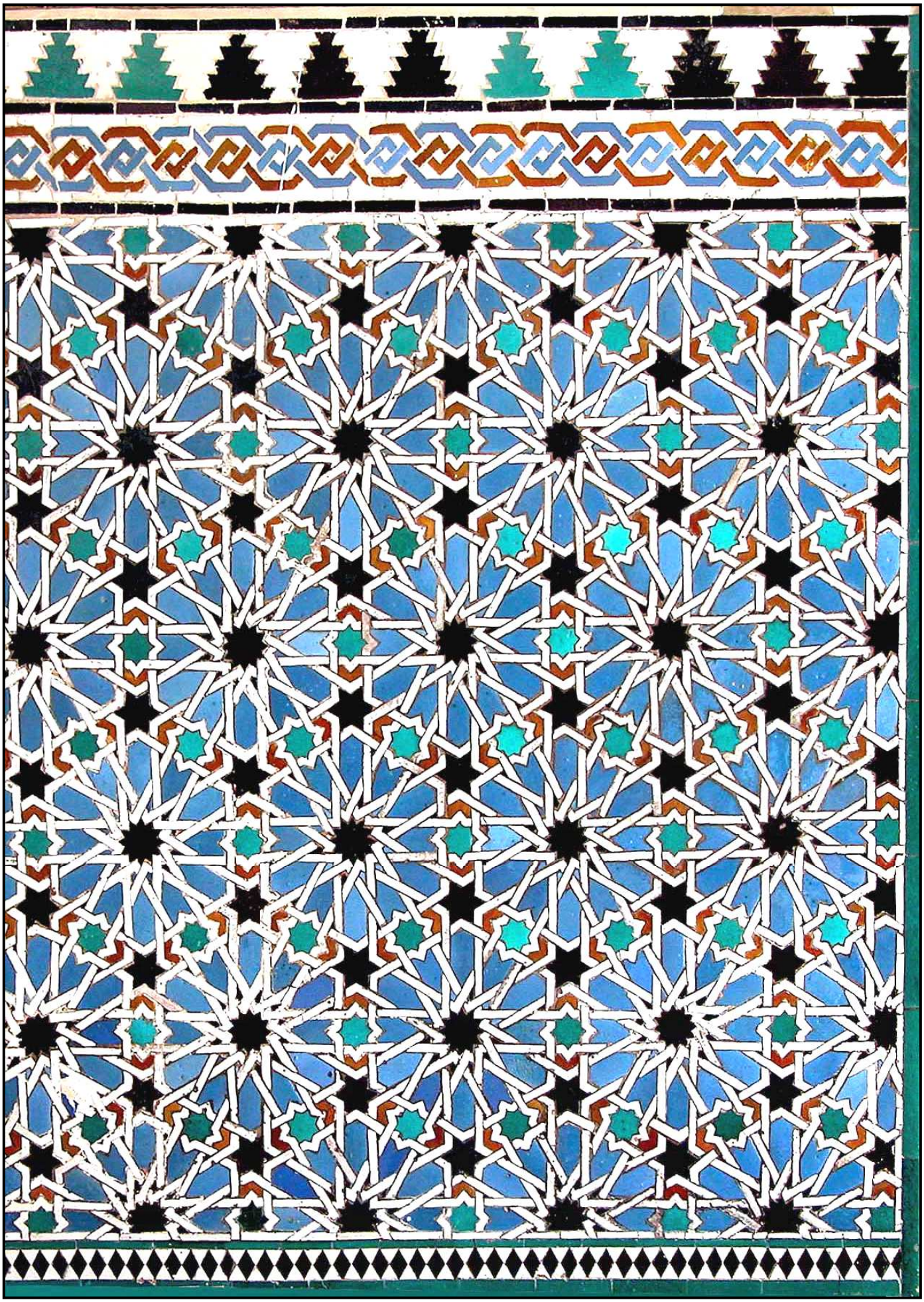
Porte d’un des mausolées ottomans de la Muradeye de Bursa.

Système convergent hexagonal :

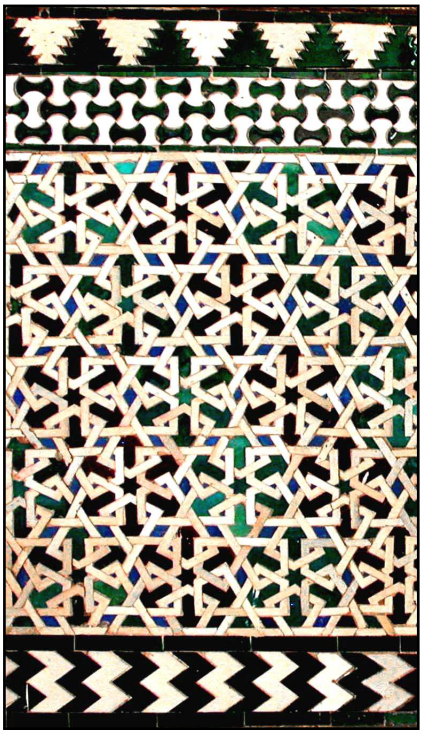




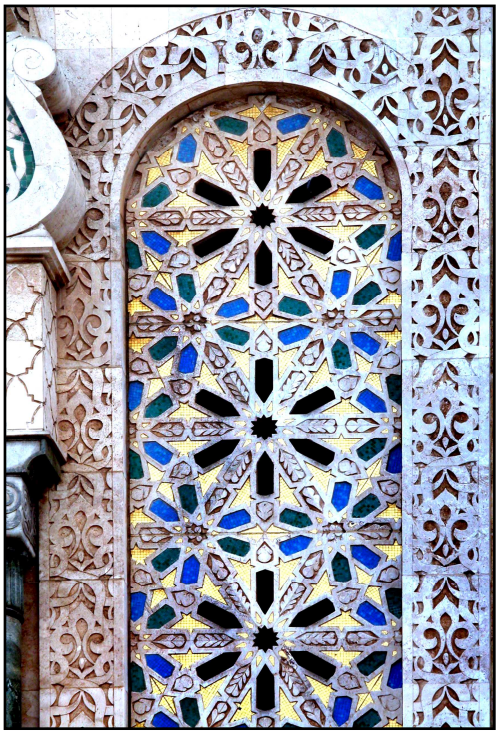
Entrelacs de cuivre formant des étoiles à douze sur le portail du palais royal de Fès.



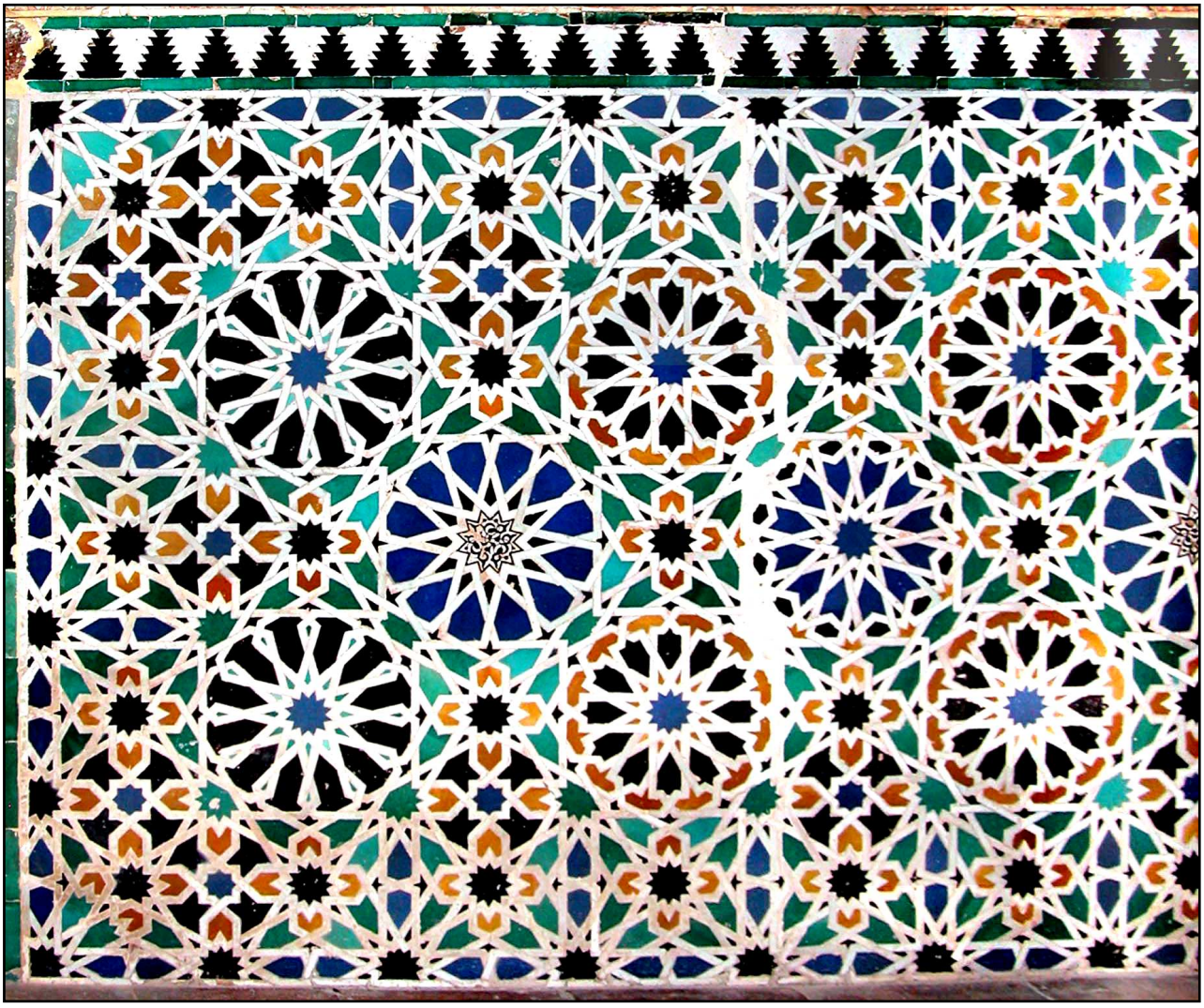
Alcazar de Séville, patio de las Doncellas.



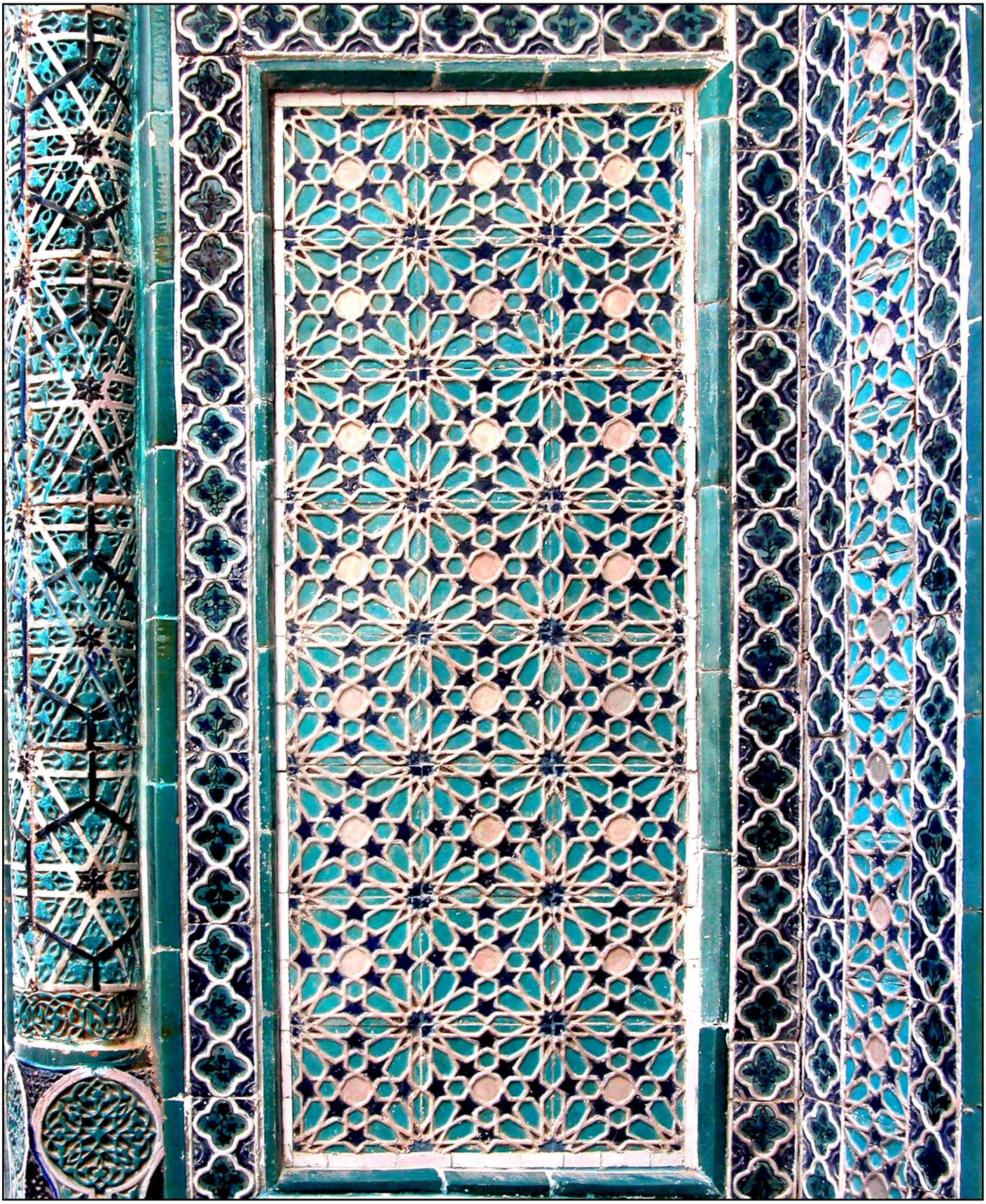
Alcazar de Séville.



Trois étoiles à douze forment une étoile à neuf à la mosquée Hassan II de Casablanca.

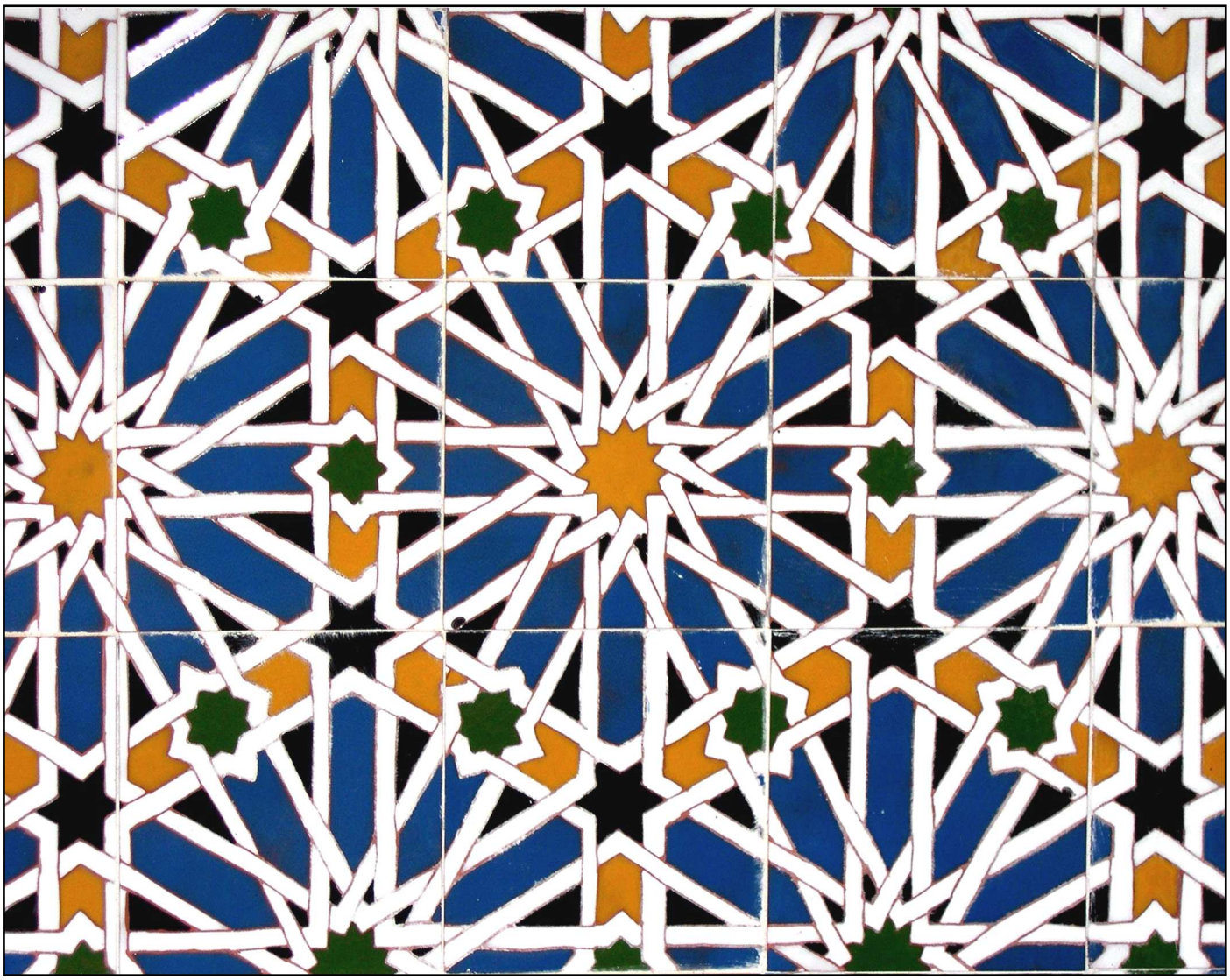


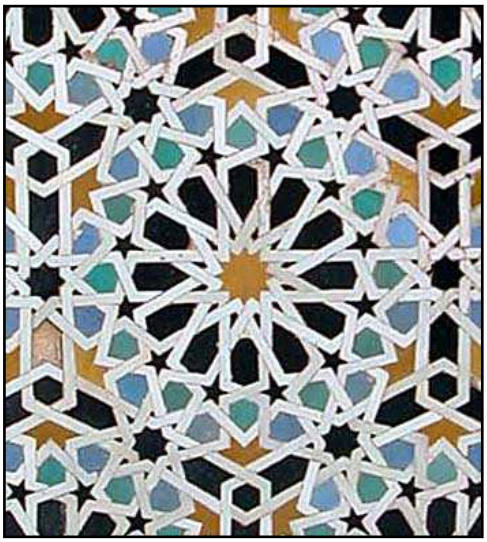
Composition d'étoiles à douze à Alhambra de Grenade.

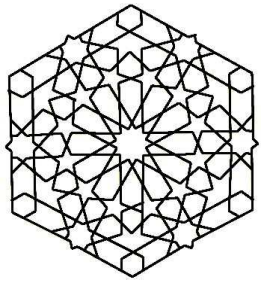


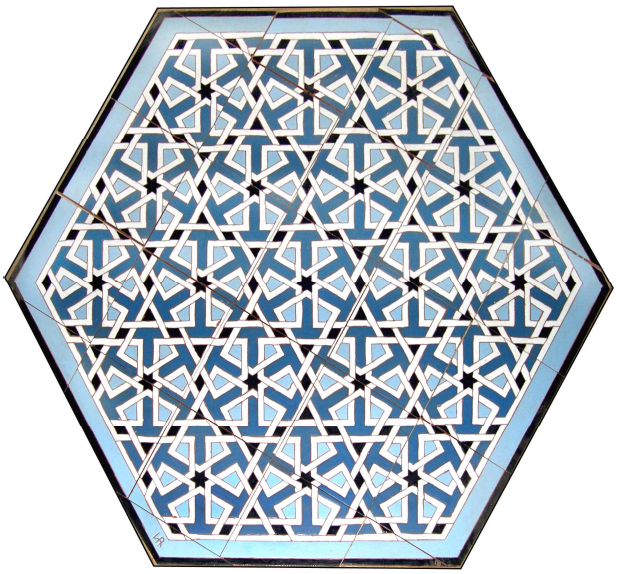
Pistach du mausolée de Khodja Akhmad à Shah-I-Zinda, nécropole de Samarcande.

Le nombre des axes de symétrie est un multiple de trois.

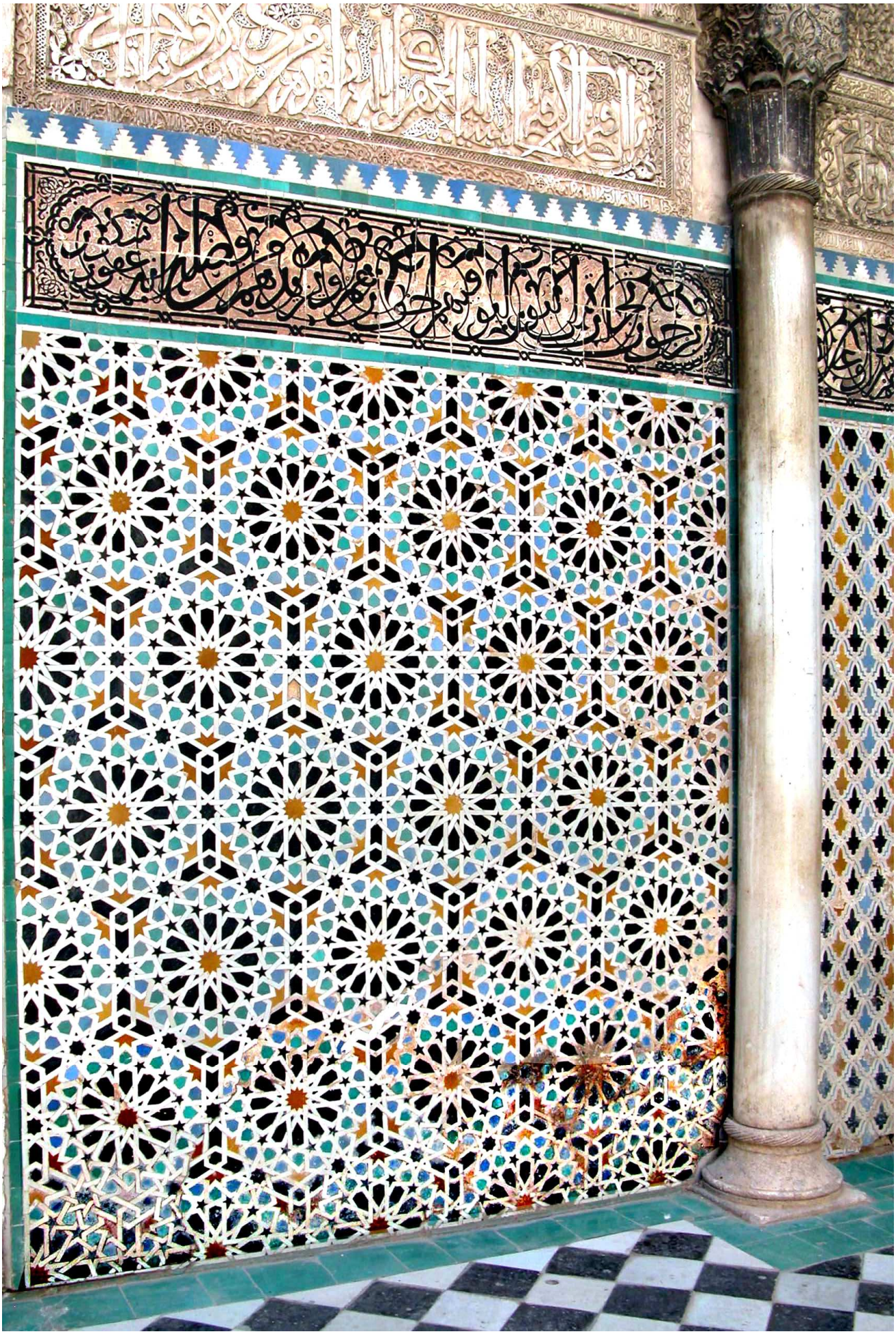






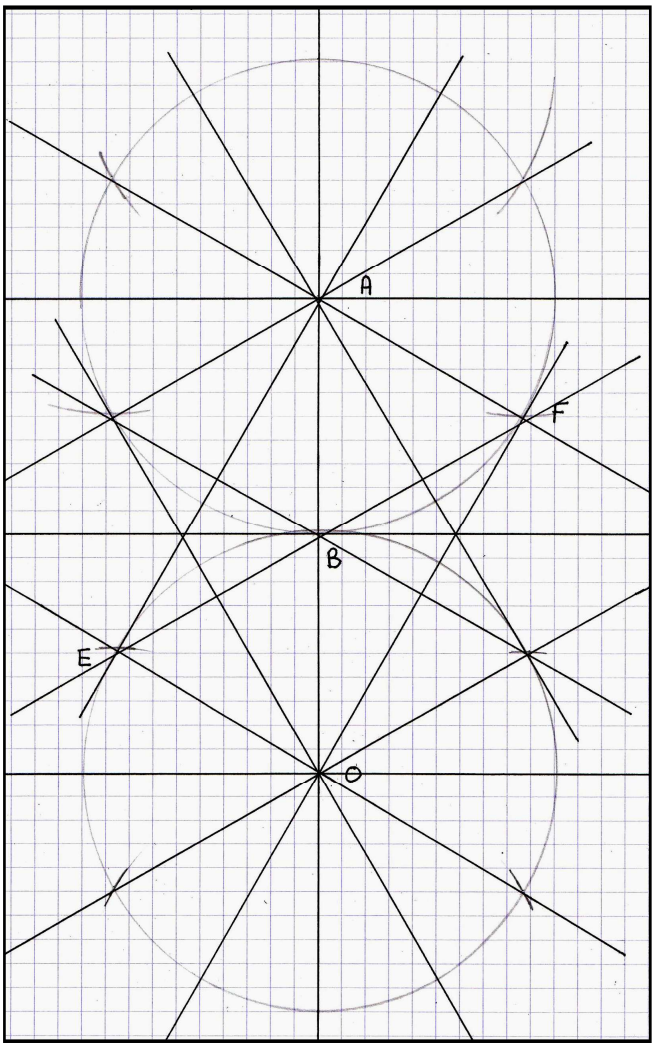


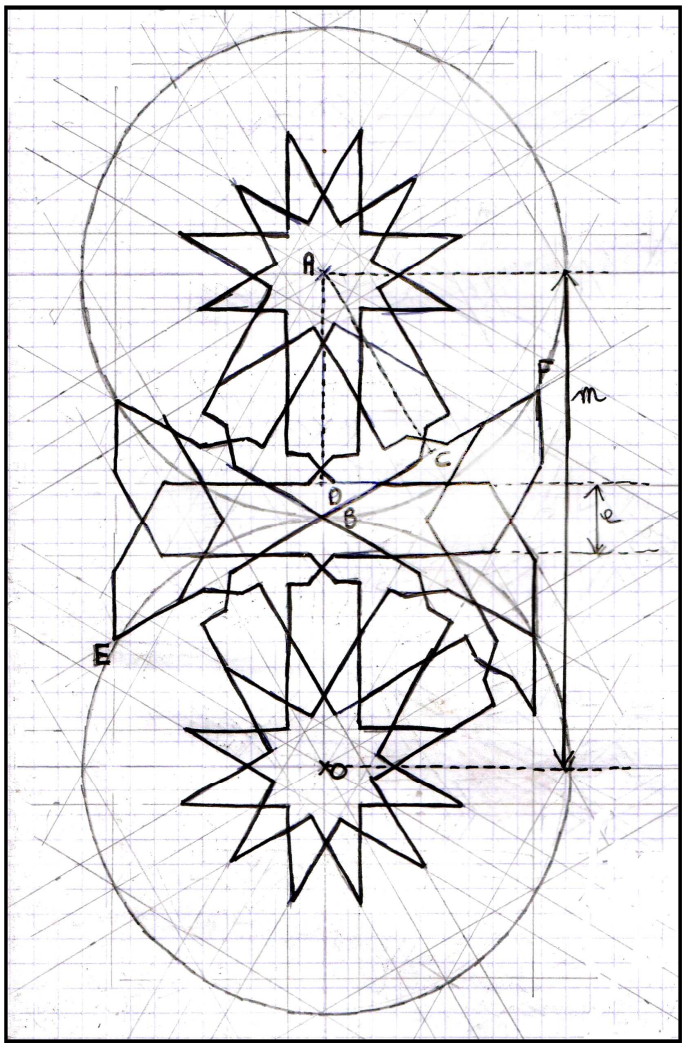
Plus de sceaux ni de saft : il faut définir une nouvelle maille ; ce sera la distance entre deux étoiles à douze.



Médersa Attarine de Fès.

- *Calcul de la maille :*





AO = m pour utiliser la méthode de dessin, il faut pouvoir calculer la largeur de la bande en fonction de **m**.

Nous connaissons la maille.

$AO = m$ distance de deux étoiles

Nous avons BD est une demi bande

$BD = \frac{e}{2}$

$BD = AB - AD$

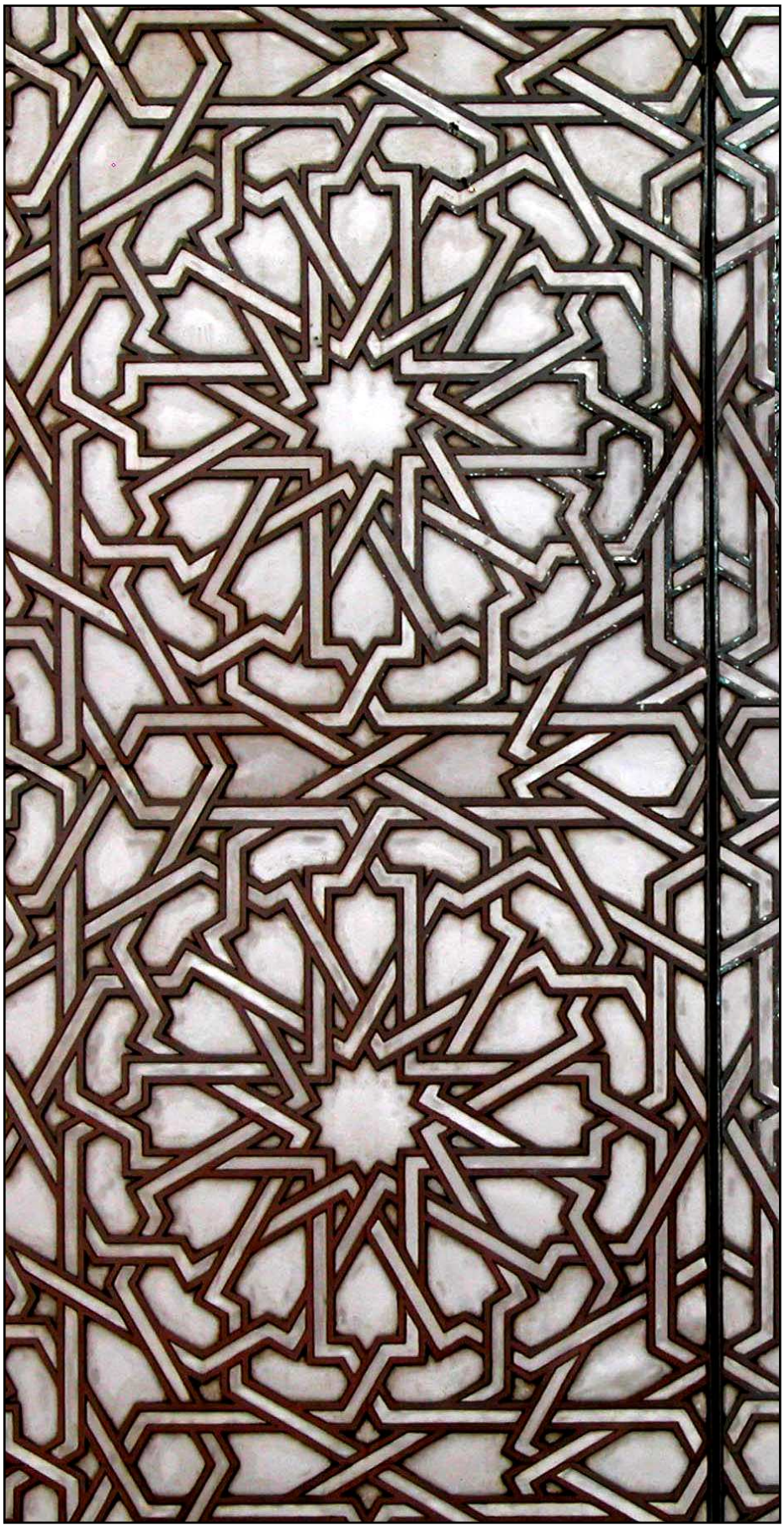
$BD = AB - AC$ or $AC = AB\frac{\sqrt{3}}{2}$ donc :

$\frac{e}{2} = \frac{m}{2} - \frac{m}{2} \cdot \frac{\sqrt{3}}{2}$ donc

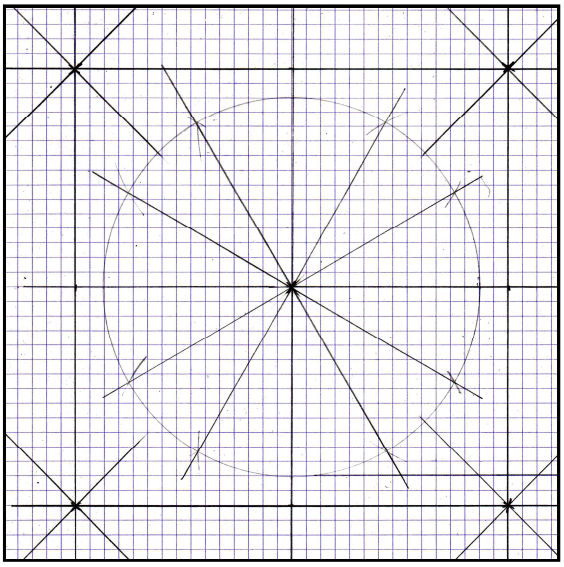
$$e = m\left(\frac{2-\sqrt{3}}{2}\right)$$

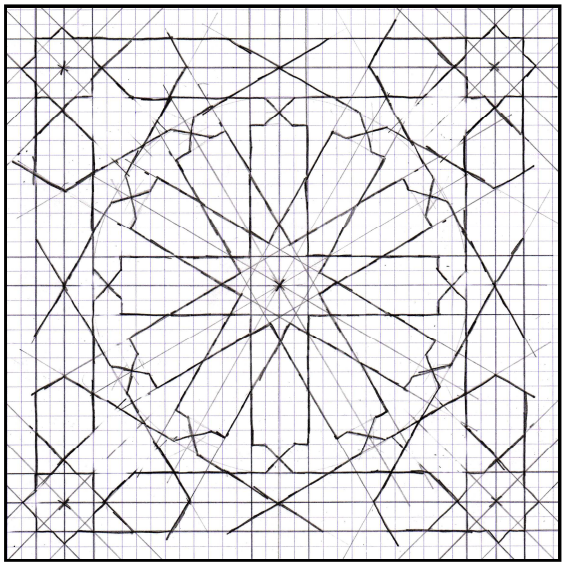
$$\boxed{e = m \times 0{,}134}$$

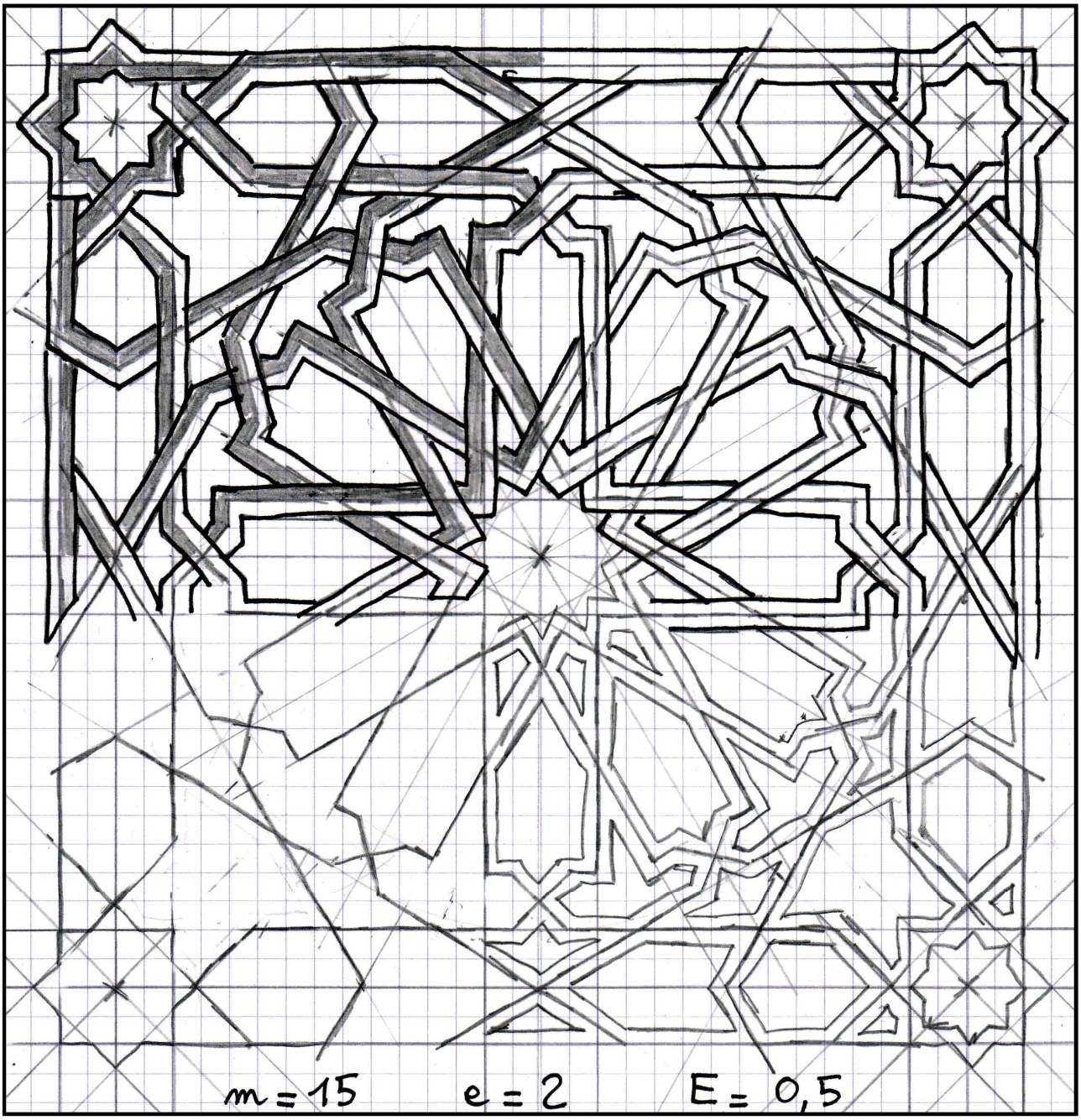
- *Construction des entrelacs :*



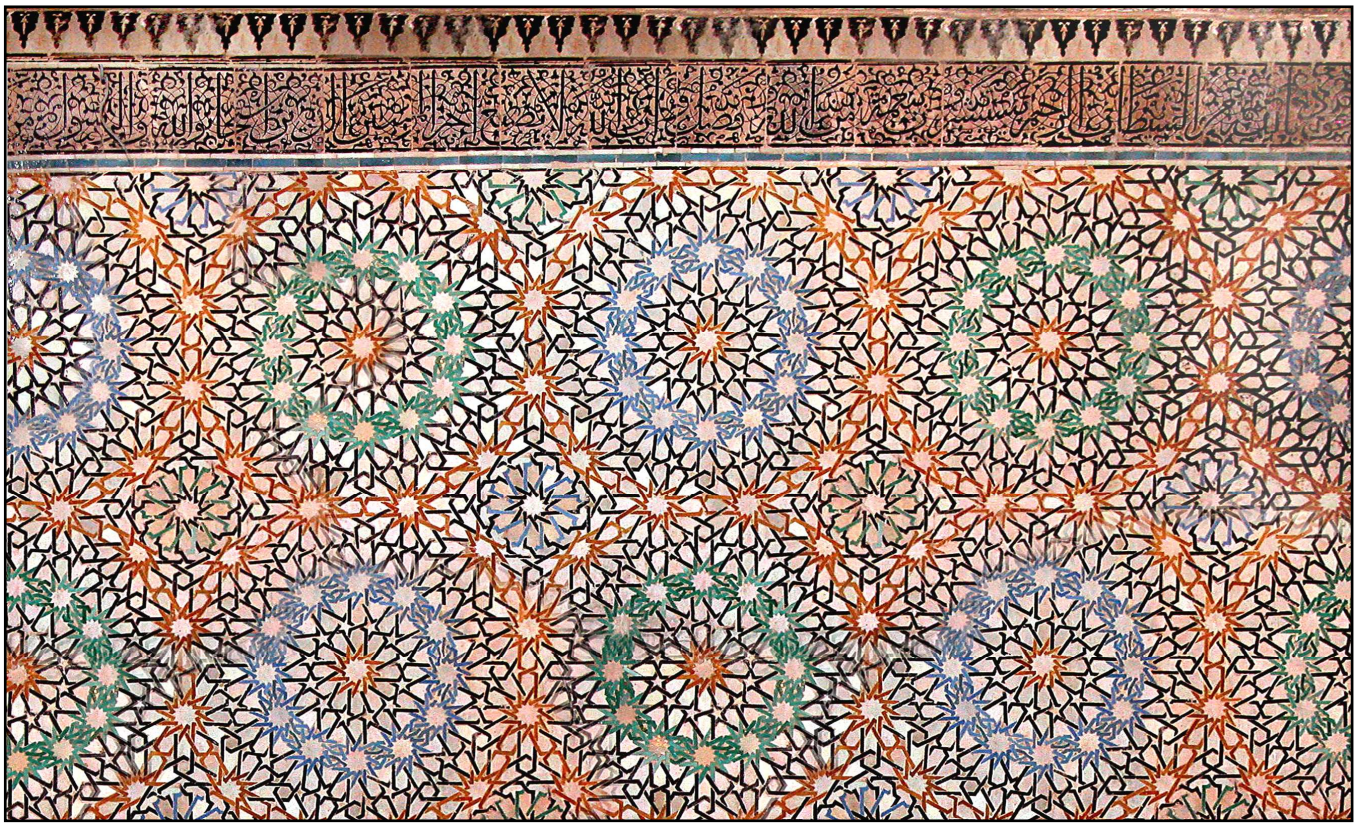
Porte de titane de la mosquée Hassan II.





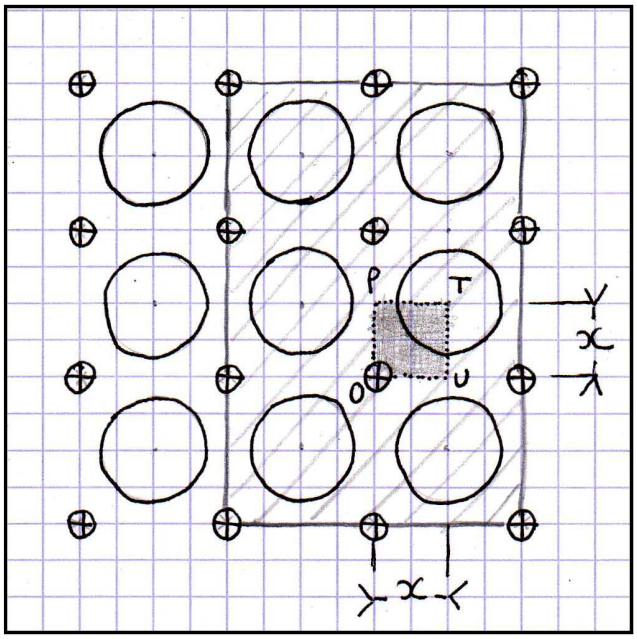

m = 15
e = 2
E = 0,5

- *Exemple de construction :*



Tombeaux Saadiens à Marrakech.

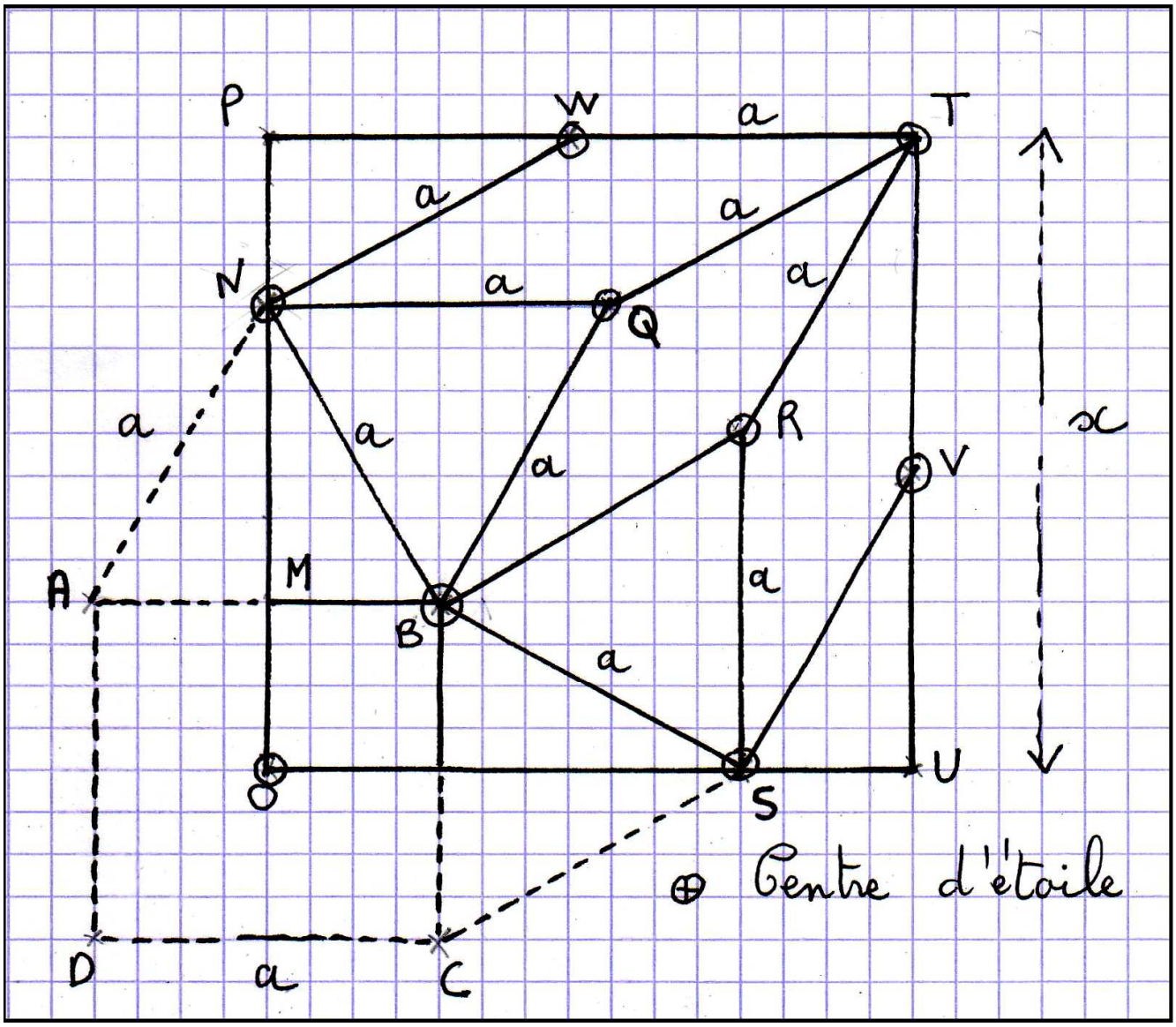
Sur cet exemple, ce sont les entrelacs qui sont colorés. Toutes les étoiles de la construction sont des étoiles à douze ainsi que les satellites.



L'étude ci contre permet de tracer le carré minimal de mesure **x** ; le choix de la valeur de **x**, sous multiple des dimensions du cadre, permet de cadrer le dessin.

Pour tracer le carré minimal **(OPTU)**, il faut construire :

- Le carré (ABCD)
- Les quatre triangles équilatéraux ayant un sommet en B.
- Le losange (BQTR) qui détermine la position du point T.
- Le carré minimal (OPTU).
- Les centres S, V, W et N. les centres des étoiles sont ainsi tous définis.
- La maille cachée **ON = m .**



a = mesure du côté des triangles équilatéraux et du carré (ABCD)

$MN = \frac{a\sqrt{3}}{2}$ $\quad MO = \frac{a}{2}$ $\quad PN = \frac{a}{2}$

$ON = m = \frac{a\sqrt{3}}{2} + \frac{a}{2}$ donc $a = m(\sqrt{3}-1)$

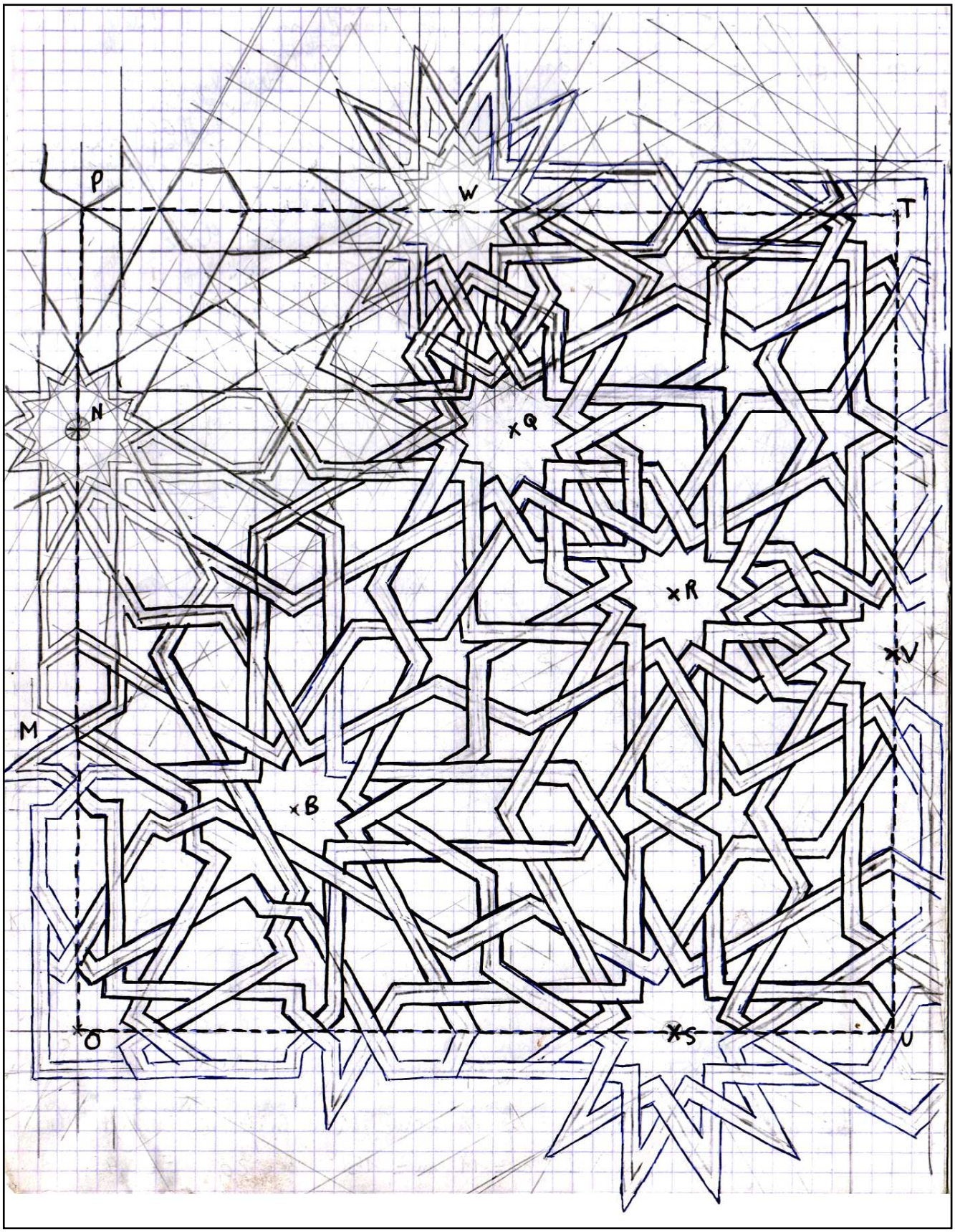
x = mesure du côté du carré minimal

$x = \frac{a}{2} + \frac{a\sqrt{3}}{2} + \frac{a}{2}$ donc $\begin{cases} x = a \times \frac{2+\sqrt{3}}{2} \\ a = m(\sqrt{3}-1) \end{cases}$

donc $m = x(\sqrt{3}-1)$

ou $x = m\,\frac{\sqrt{3}+1}{2}$

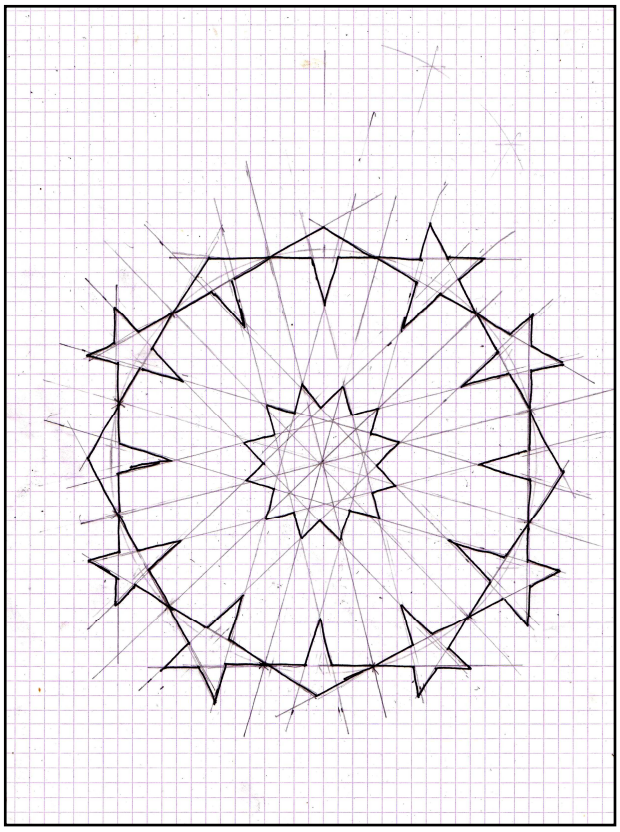
Tous les éléments permettant la construction de ce panneau sont connus.

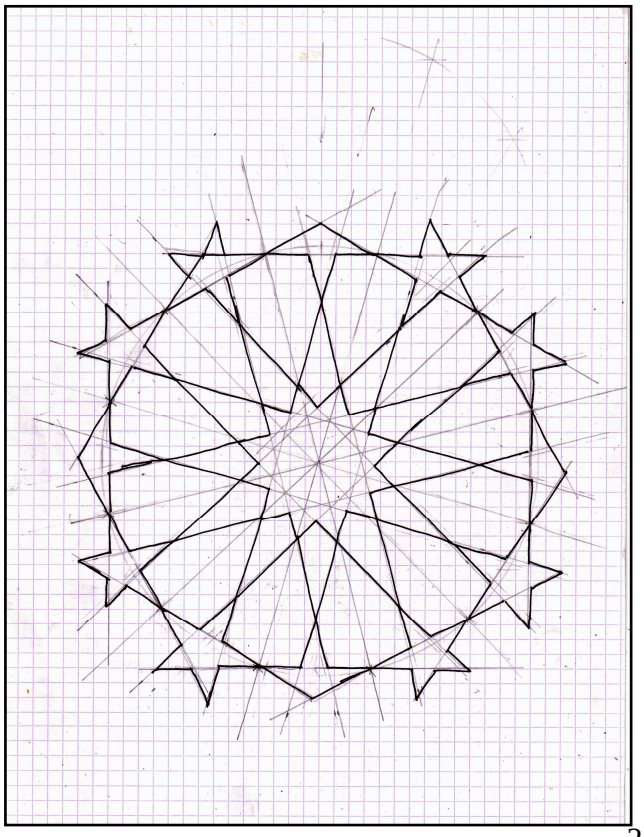


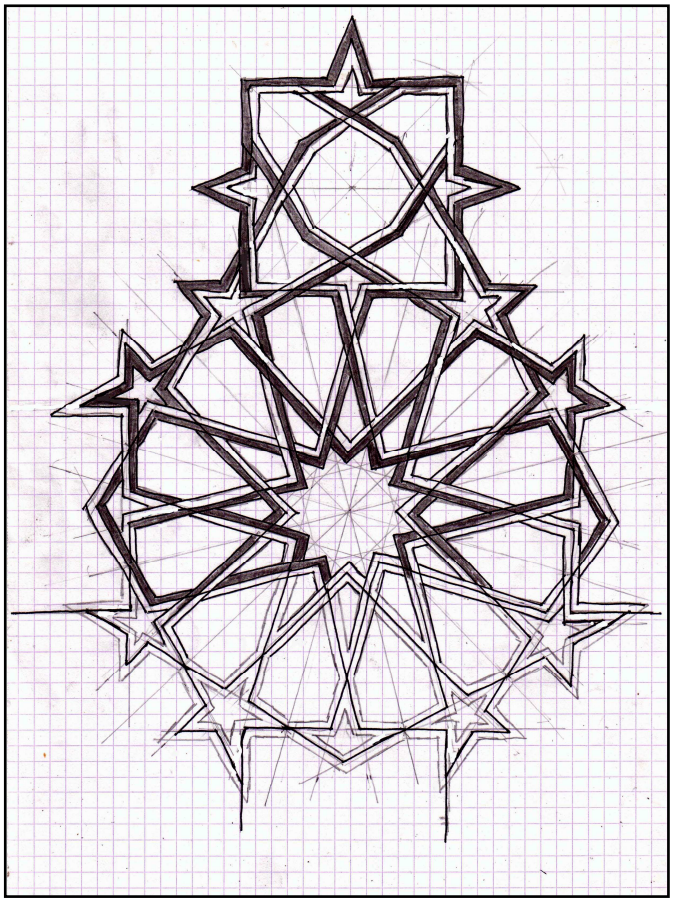
1. **x = 20,5** permet de calculer **m = 14,4** puis **a = 11** **e = 2** et **E = 0,4**
2. Construction du carré **(ABCD),** des triangles équilatéraux **(ABN)(BCS)(BSR)(NBQ)** , du losange **(BQRT)** et des points **P , U , V , W.**
3. Il suffit de terminer la construction de la manière habituelle.

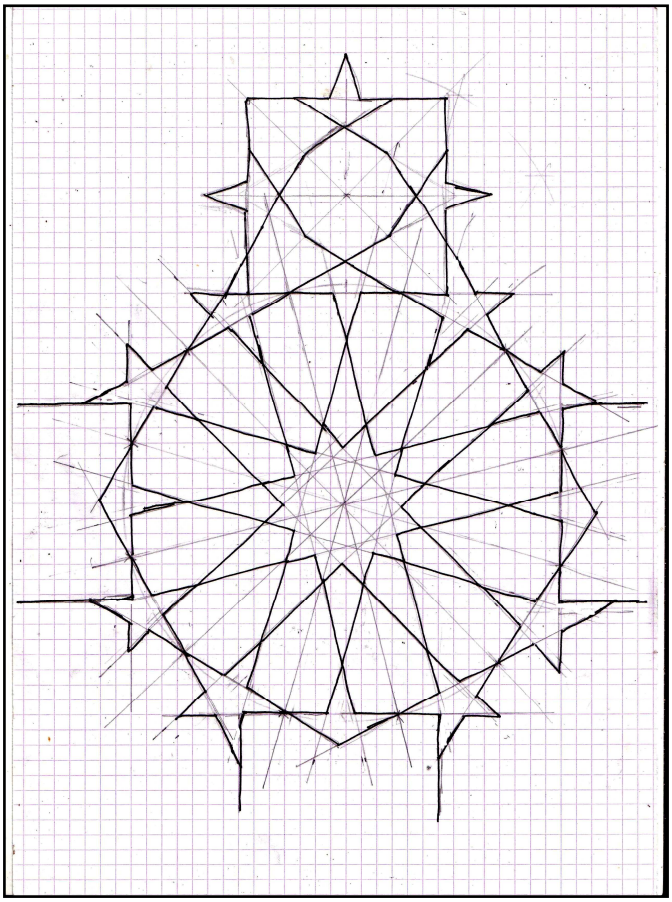
Mausolée de Moulay Ismaïl à Mekhnès.

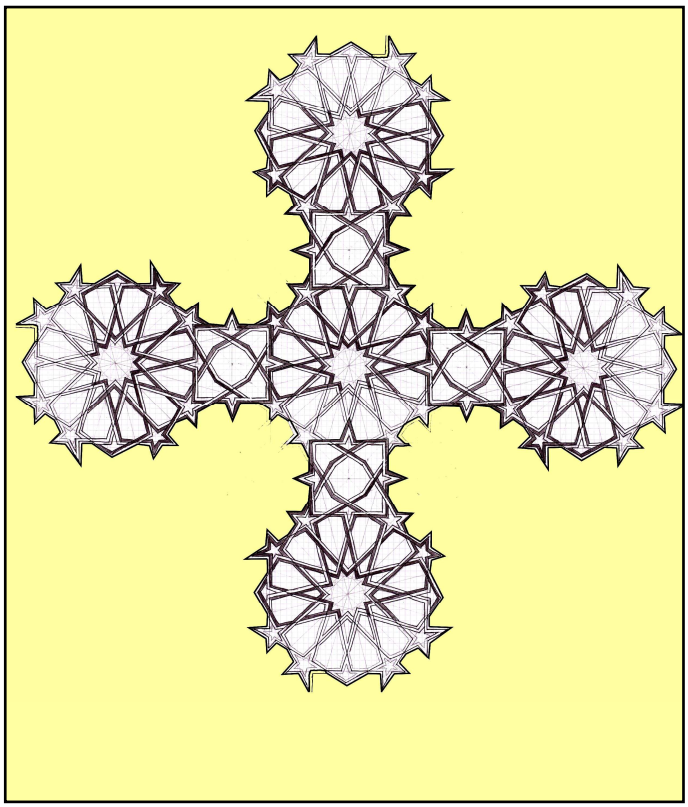
Cette porte en bronze du douzième, construite par l'architecte et mathématicien El-Cezire, provient de la mosquée de Cizre au sud est de l'Anatolie.

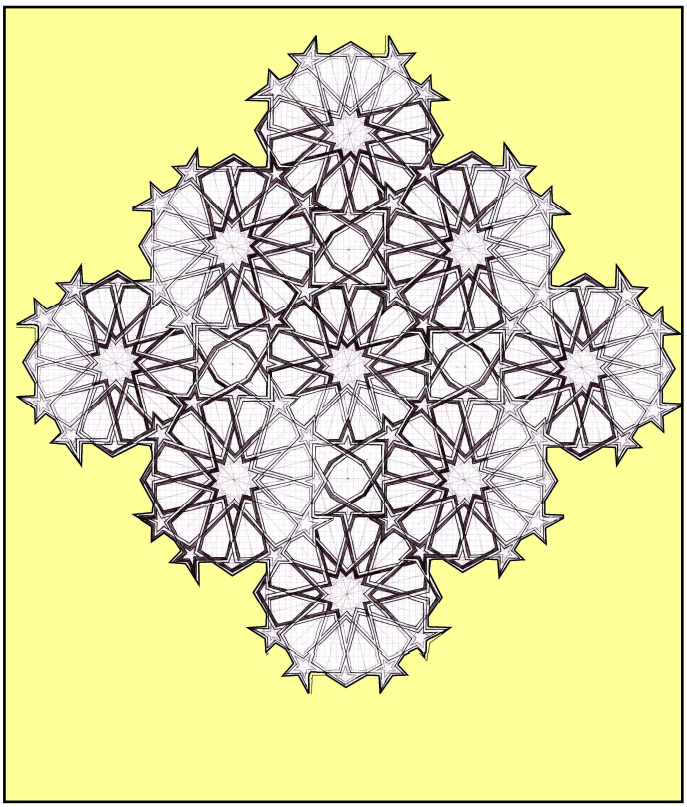


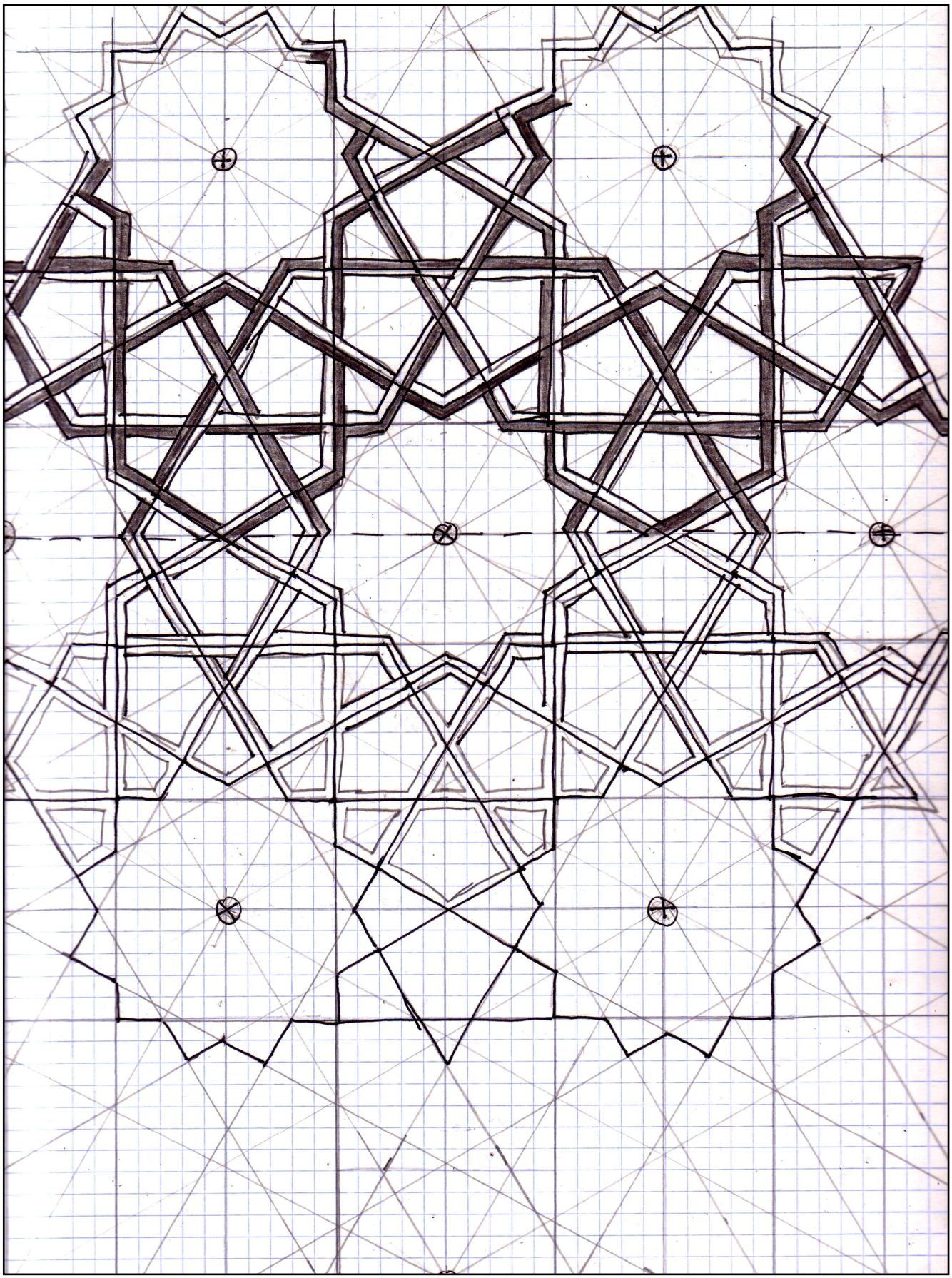


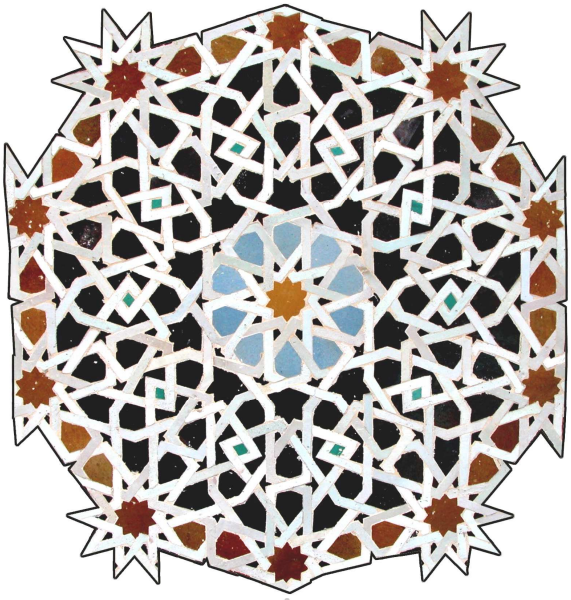


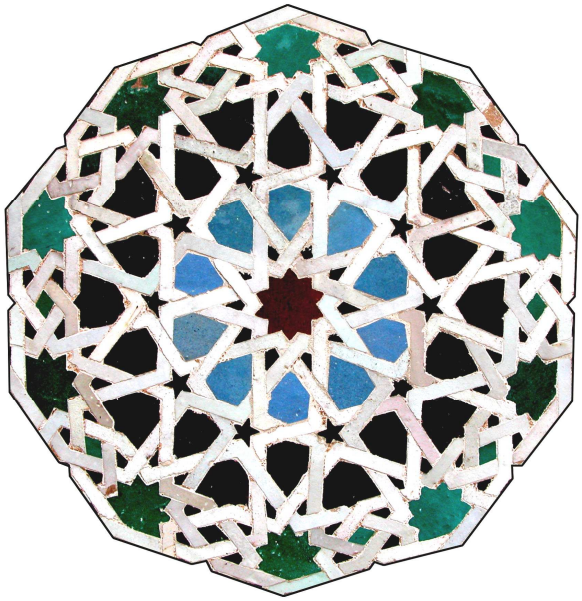




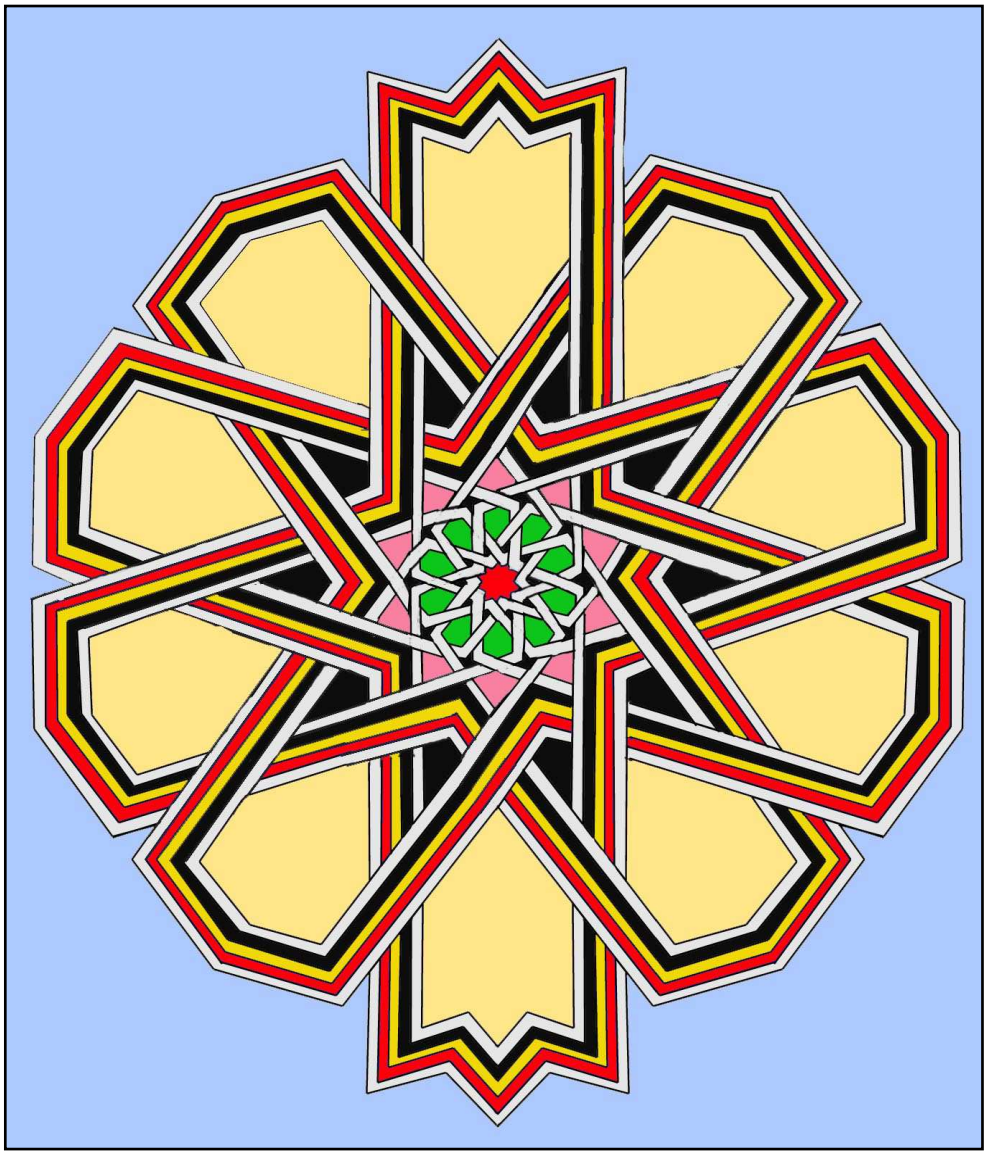


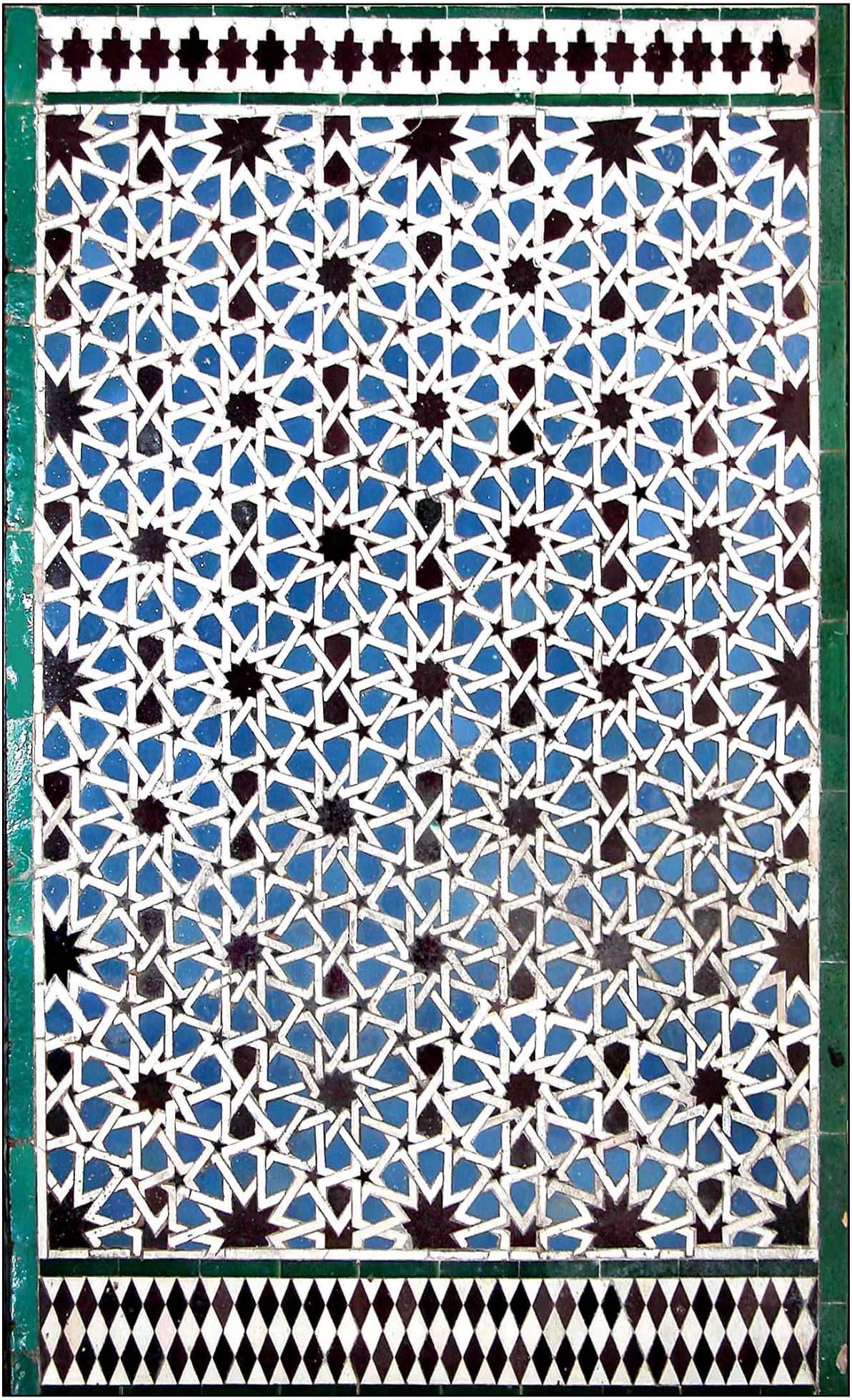




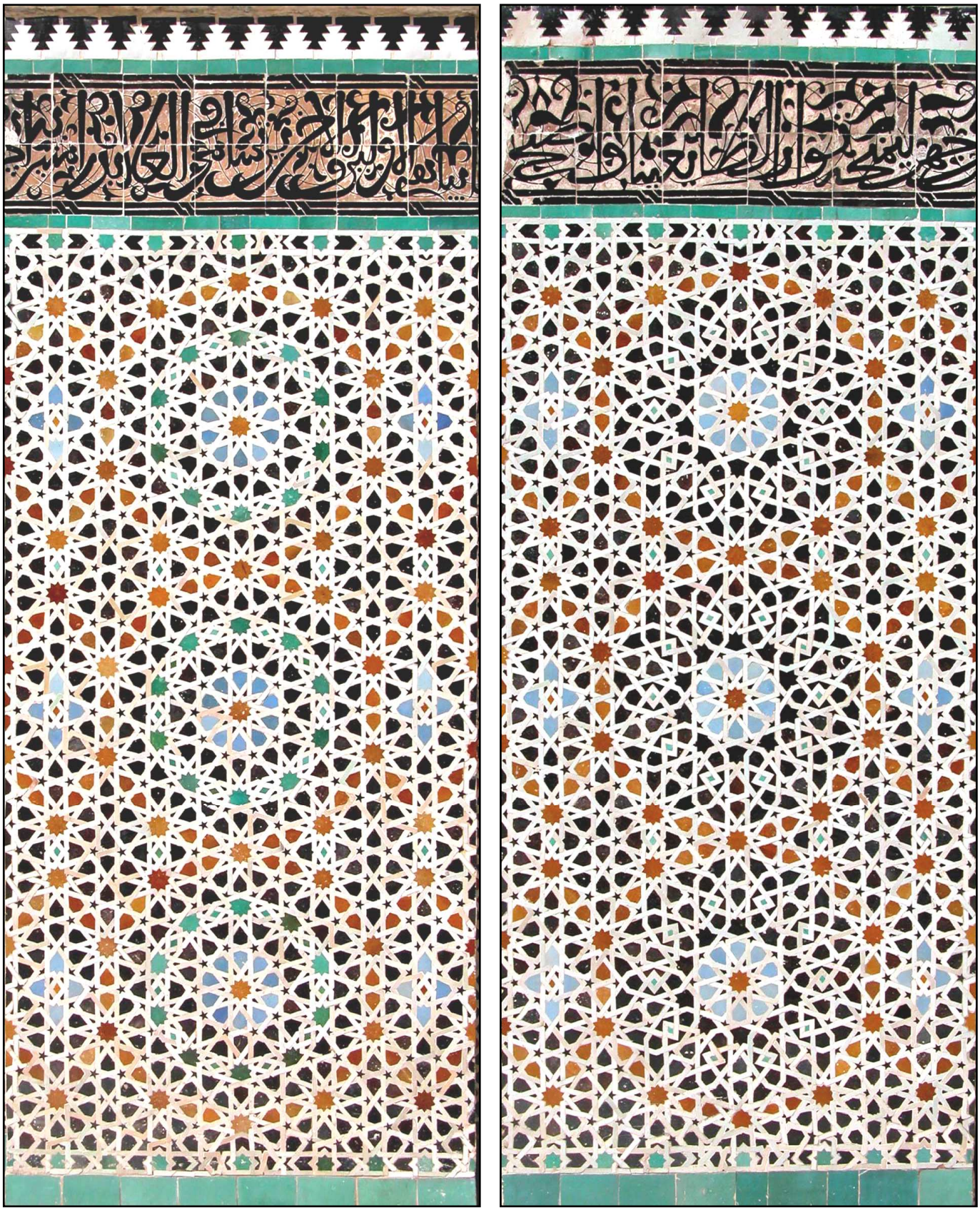


Système convergent Pentagonal.

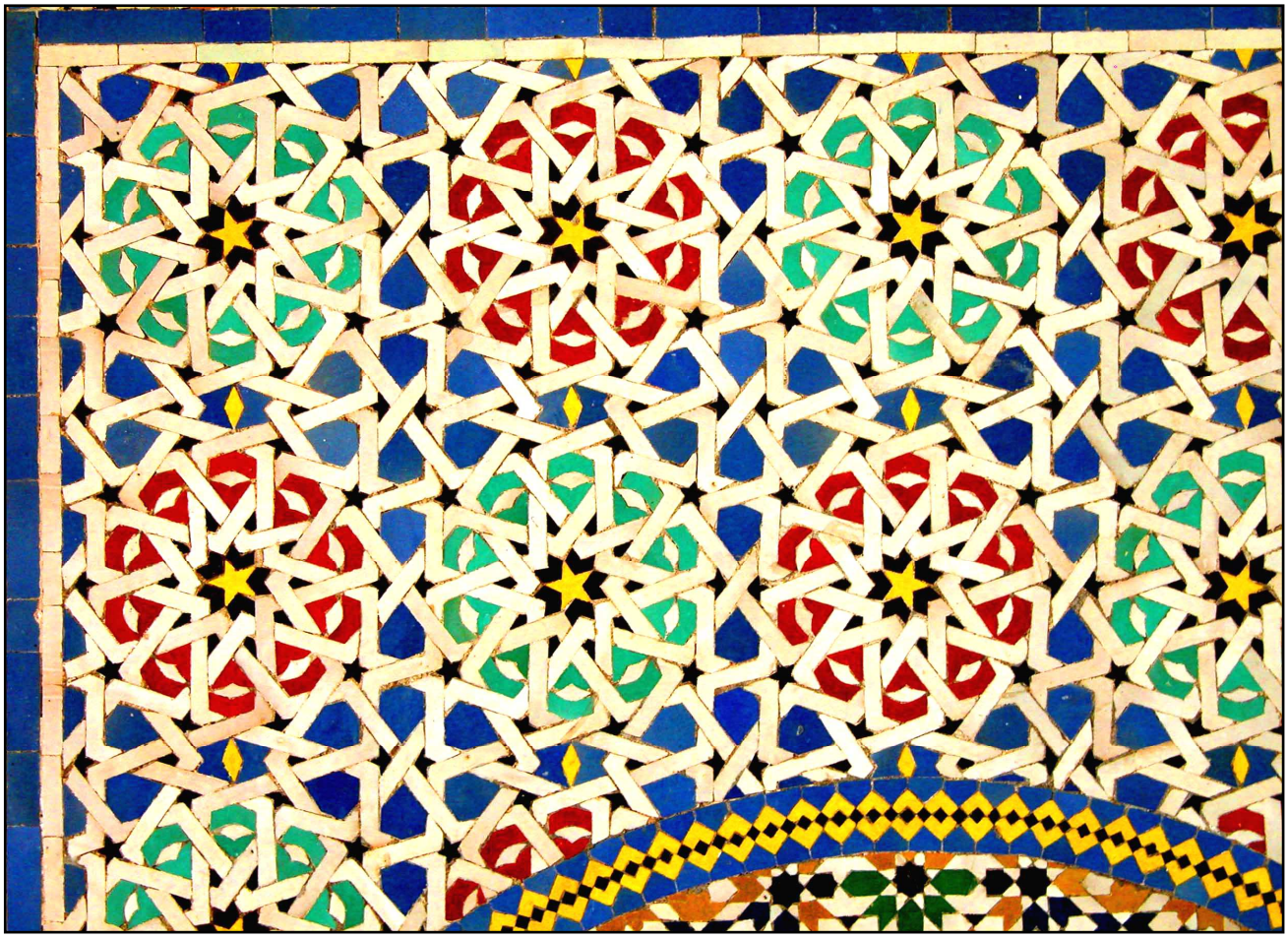




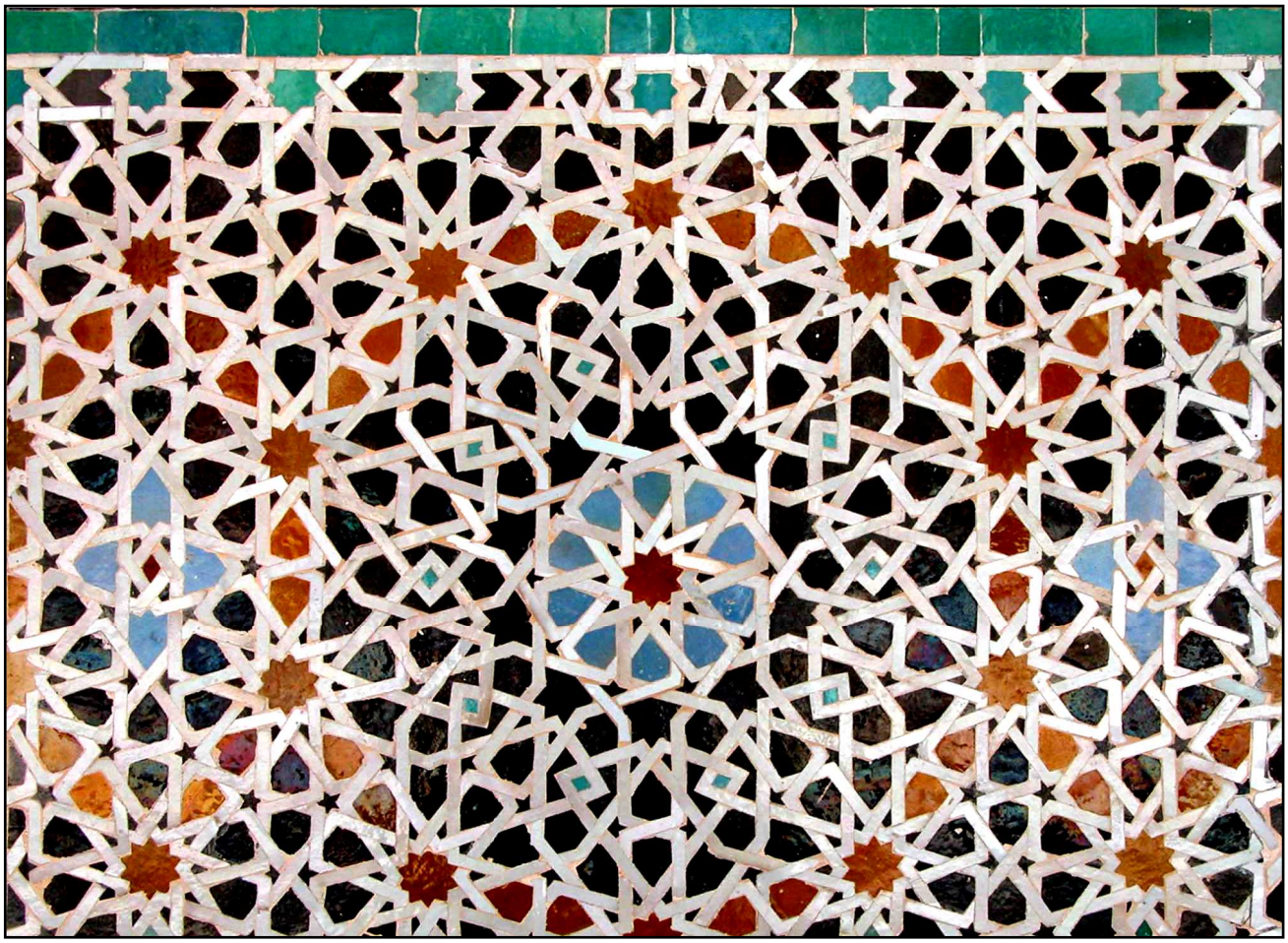
Alcazar de Séville.

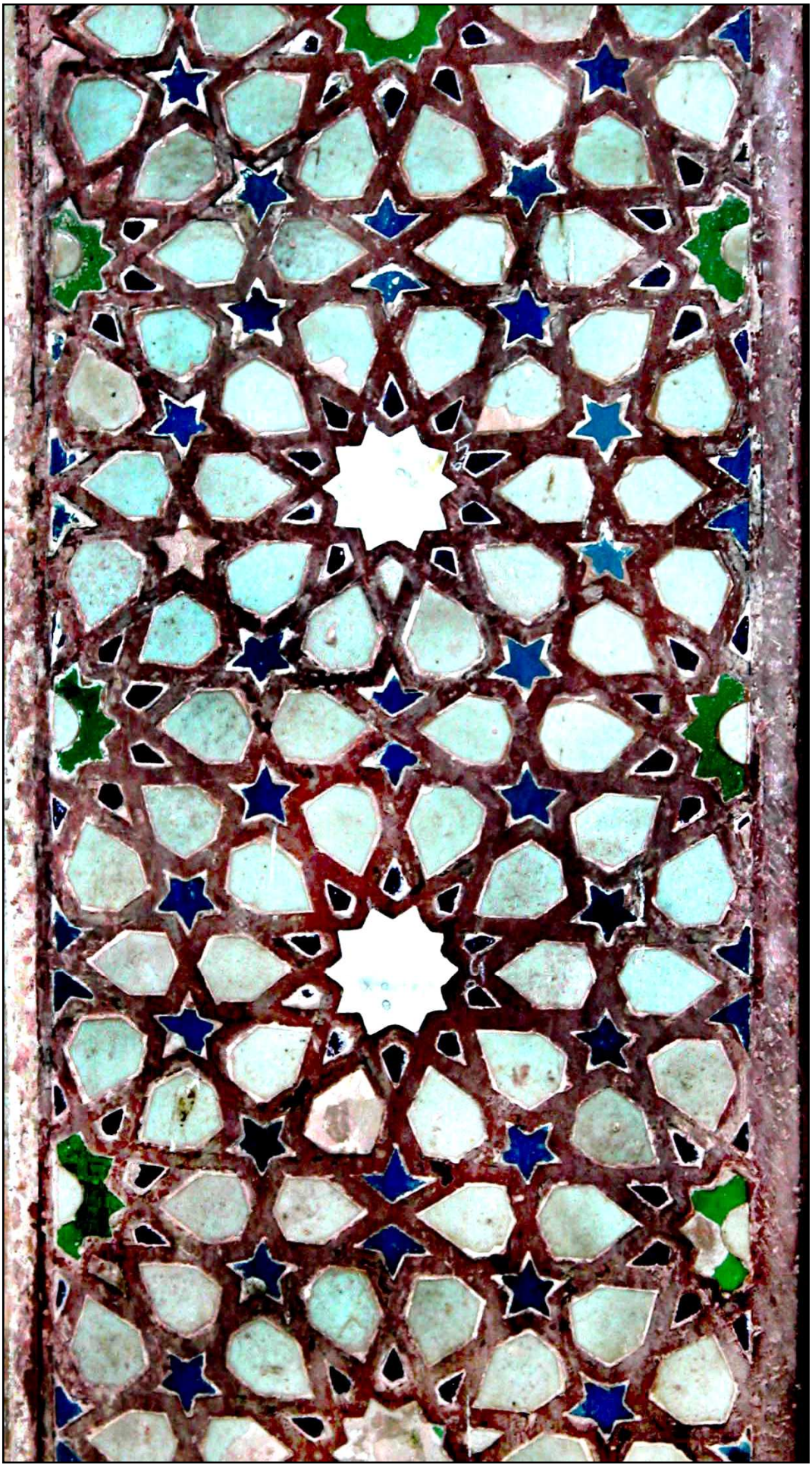


Panneaux de zelliges à la médersa Bou Inania de Fès.

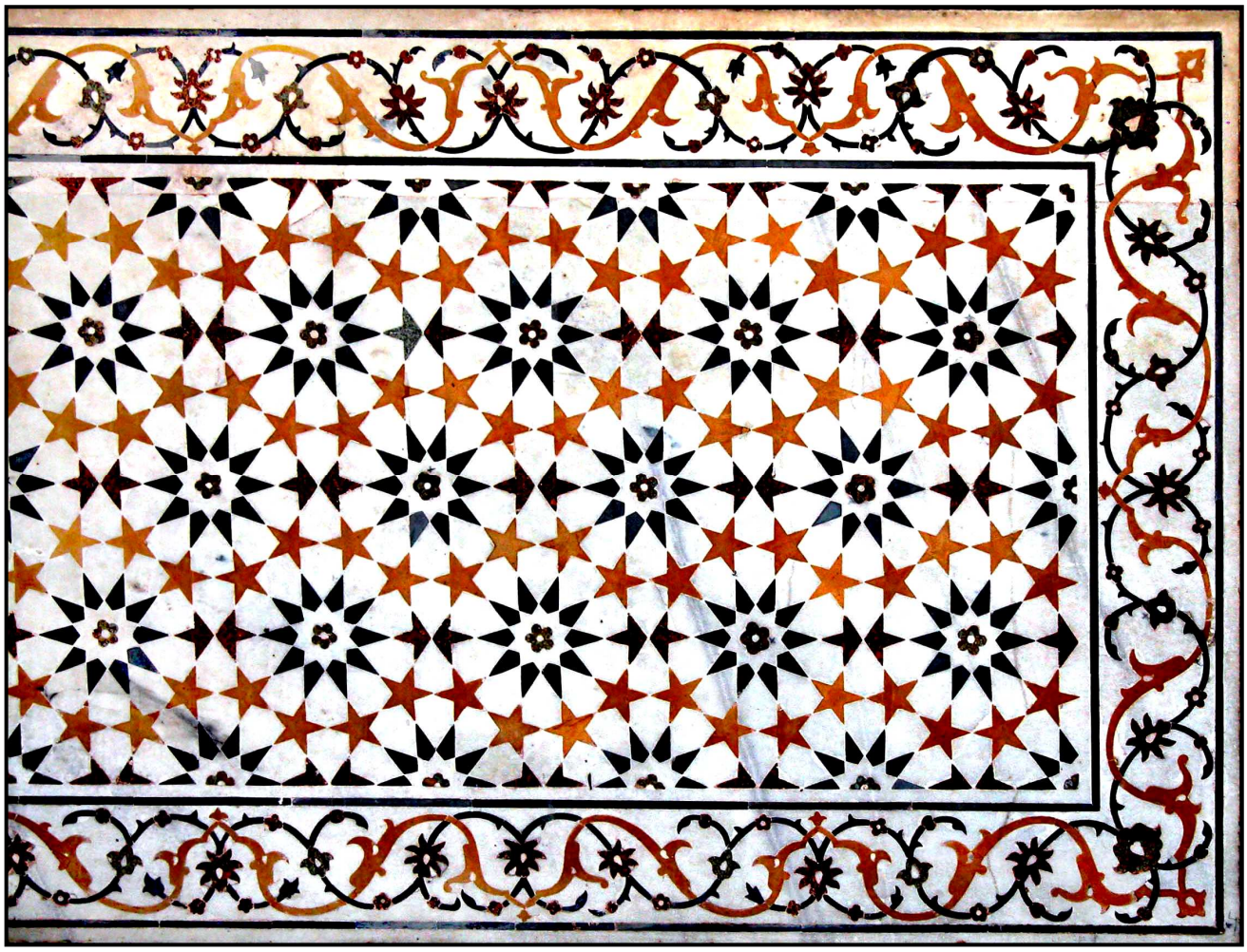


Au dessus fontaine du centre artisanal de Rabat ; panneau de zelliges à la médersa Bou Inania.

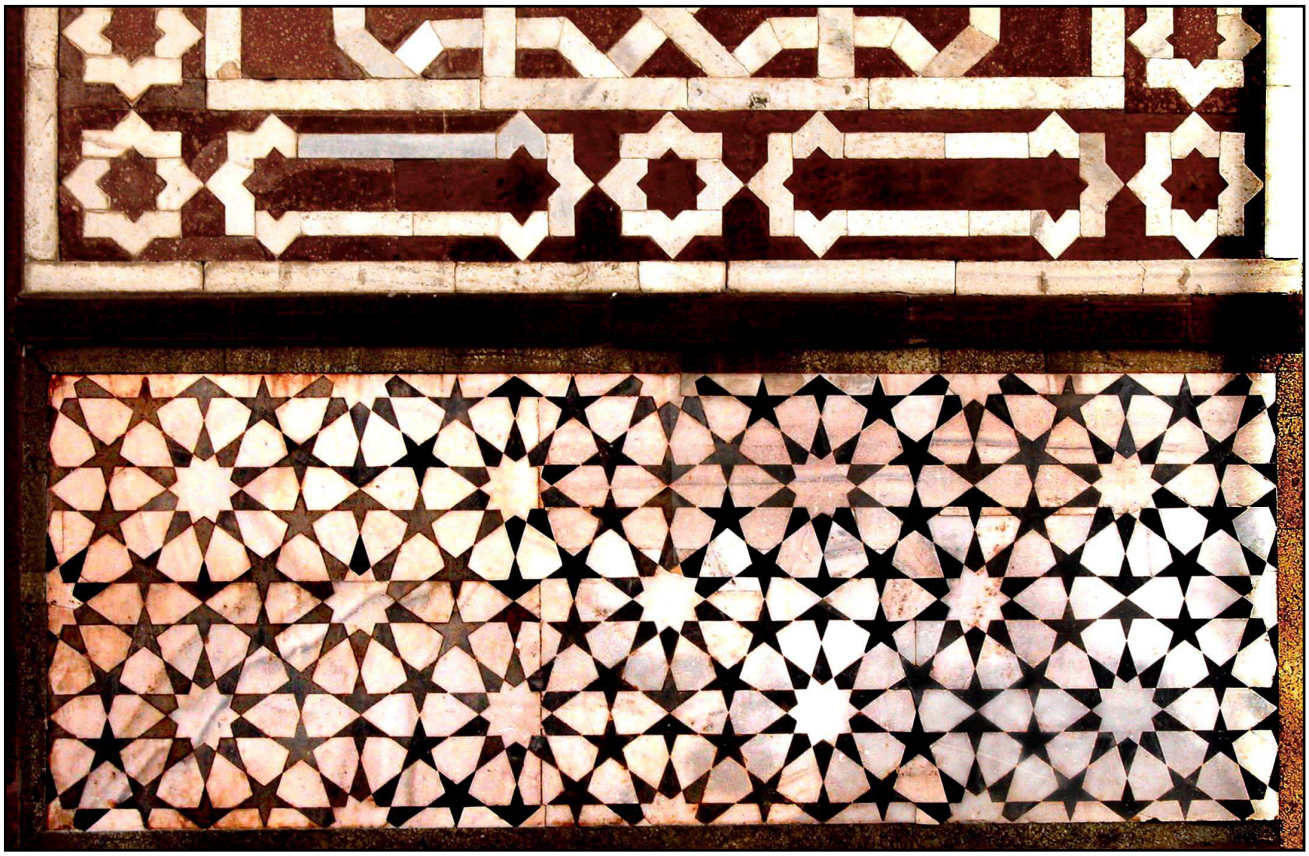




Incrustations de marbre et de pierres colorées dans du grès ; Jama Masdjid de Fatehpur Sikri.

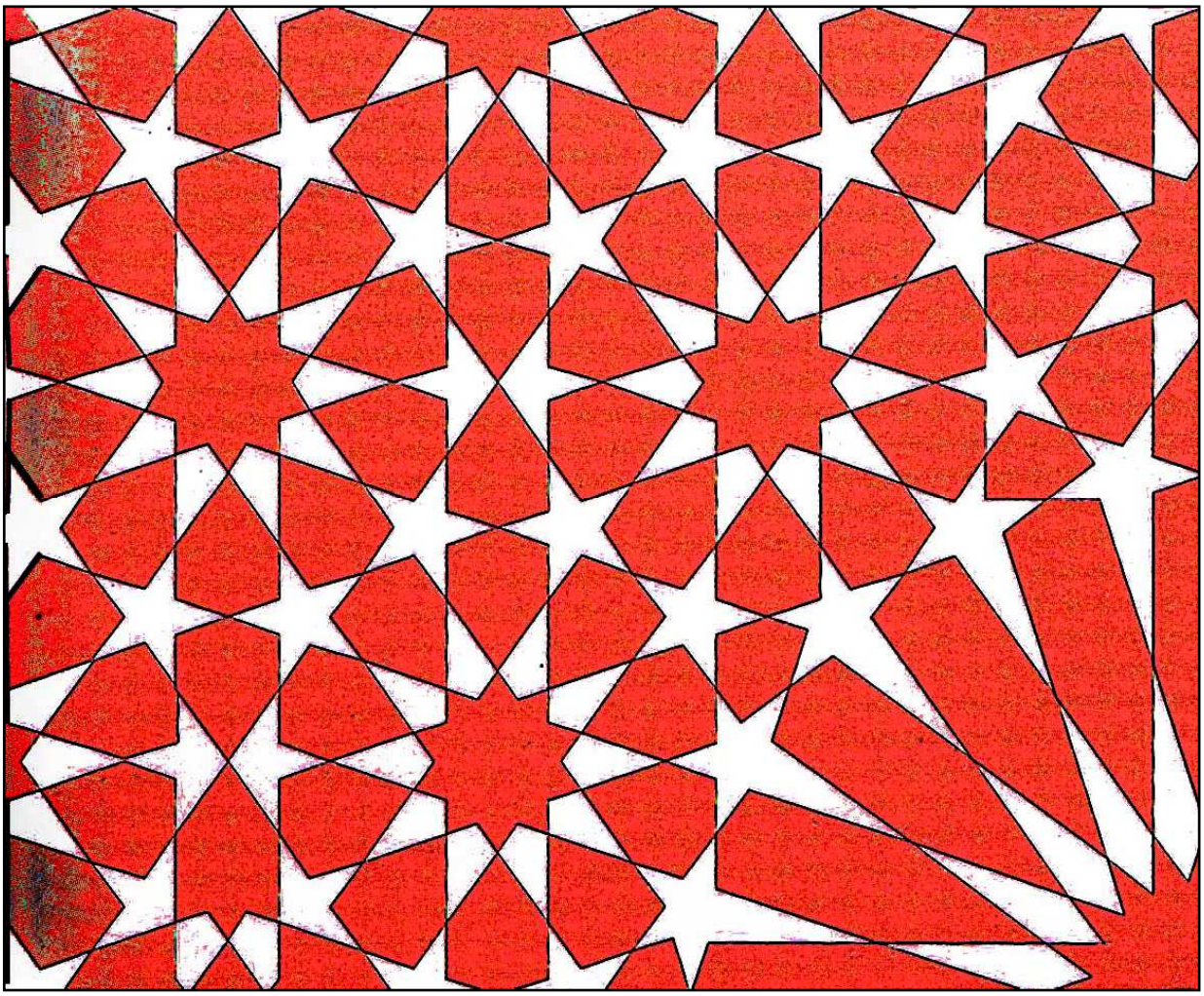


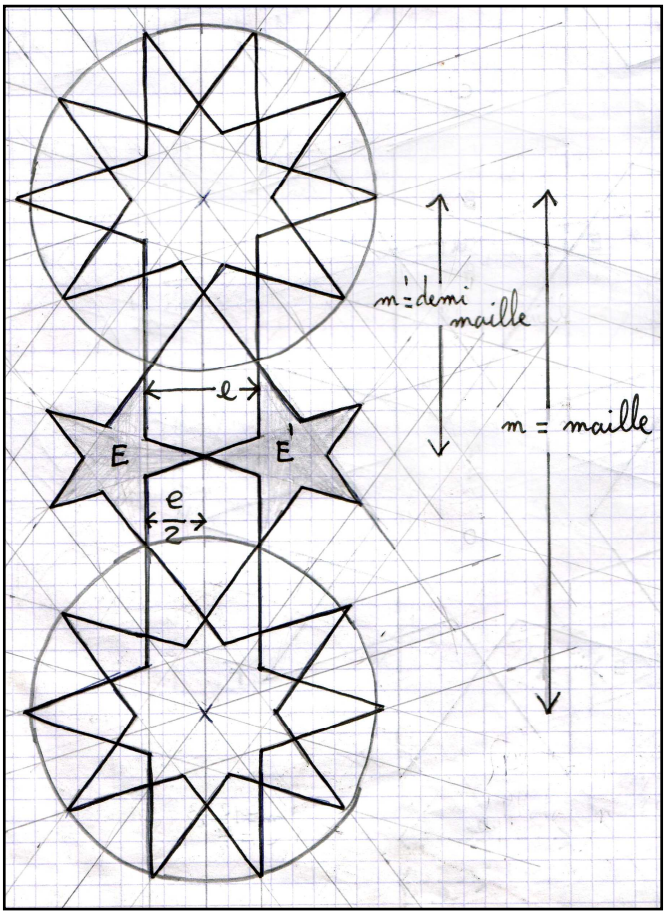
Incrustation de pietra dura au mausolée d'Itimad-ud-Daulah à Agra.



Mausolée d'Akbar à Sikandra près d'Agra.

- *Calcul de la maille et des entrelacs :*



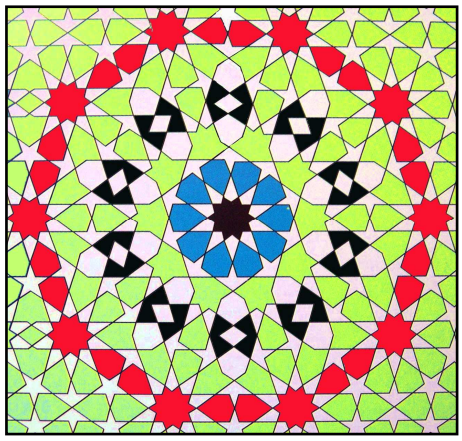


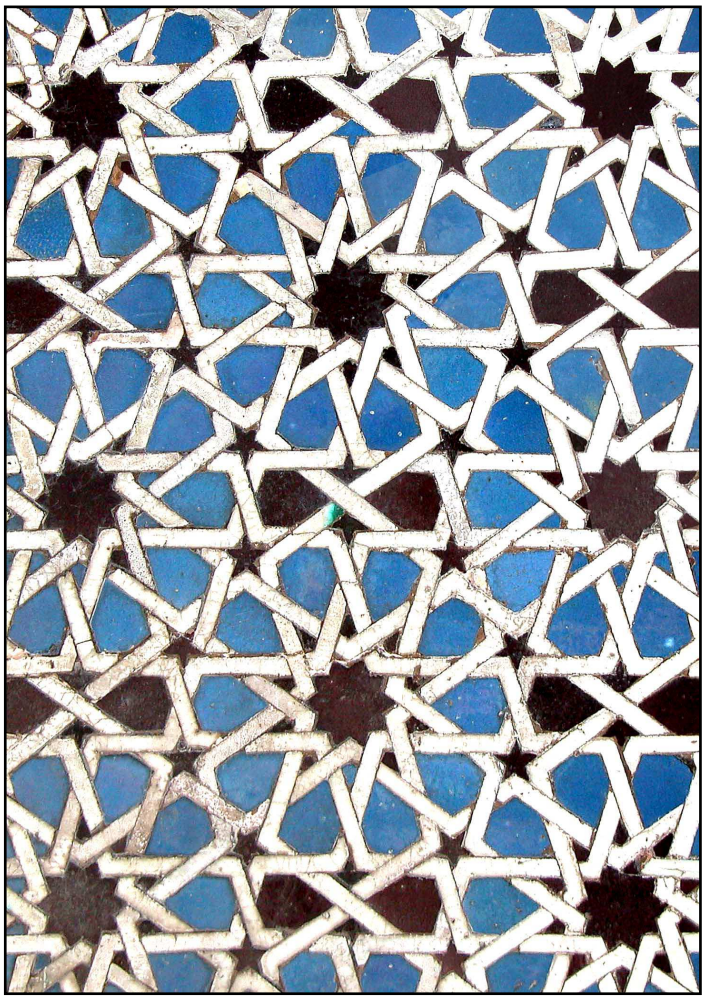
Cette structure se rencontre dans de nombreux types de systèmes convergents ayant au moins cinq axes de symétrie : la maille **m** sera définie comme la distance située entre deux étoiles à dix.

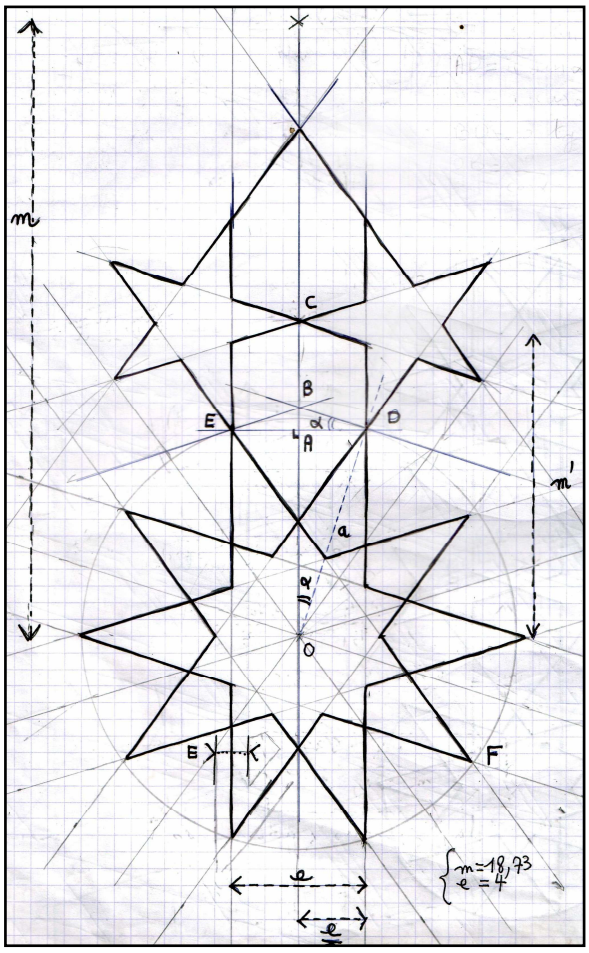
La régularité des pentagones étoilés **E** et **E'** dépend de la largeur de la bande qui sera calculée en fonction de cette exigence.

$$\begin{cases} e' = AD = a \sin\alpha \\ AO = a \cos\alpha \\ BD = a \operatorname{tg}\alpha \\ OB = \dfrac{a}{\cos\alpha} \end{cases}$$

$$\begin{cases} \sin 18° = \dfrac{-1+\sqrt{5}}{4} \\ \cos 18° = \dfrac{5+\sqrt{5}}{8} \\ \operatorname{tg} 18° = \dfrac{3\sqrt{5}-5}{5} \\ \sqrt{5} = 2{,}2360678 \end{cases}$$







$\alpha = (\widehat{OB, OD}) = 18°$

m = maille $\quad m' = \frac{1}{2}$ maille

e = bande $\quad e' = \frac{1}{2}$ bande

$CO = m' = OB + BC$

$m' = OB + BD$ (Condition de régularité de l'étoile à 5)

$$m' = \frac{a}{\cos\alpha} + a \operatorname{tg}\alpha$$

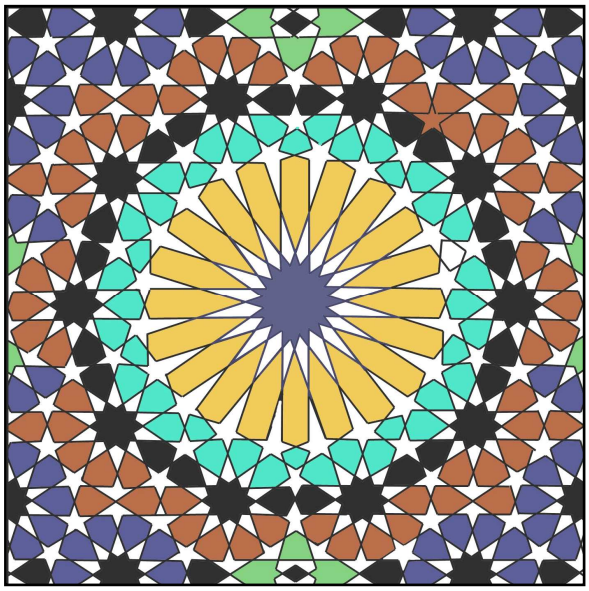
$$m' = a\left(\frac{1}{\cos\alpha} + \operatorname{tg}\alpha\right) = \frac{e'}{\sin\alpha}\left(\frac{1}{\cos\alpha} + \operatorname{tg}\alpha\right)$$

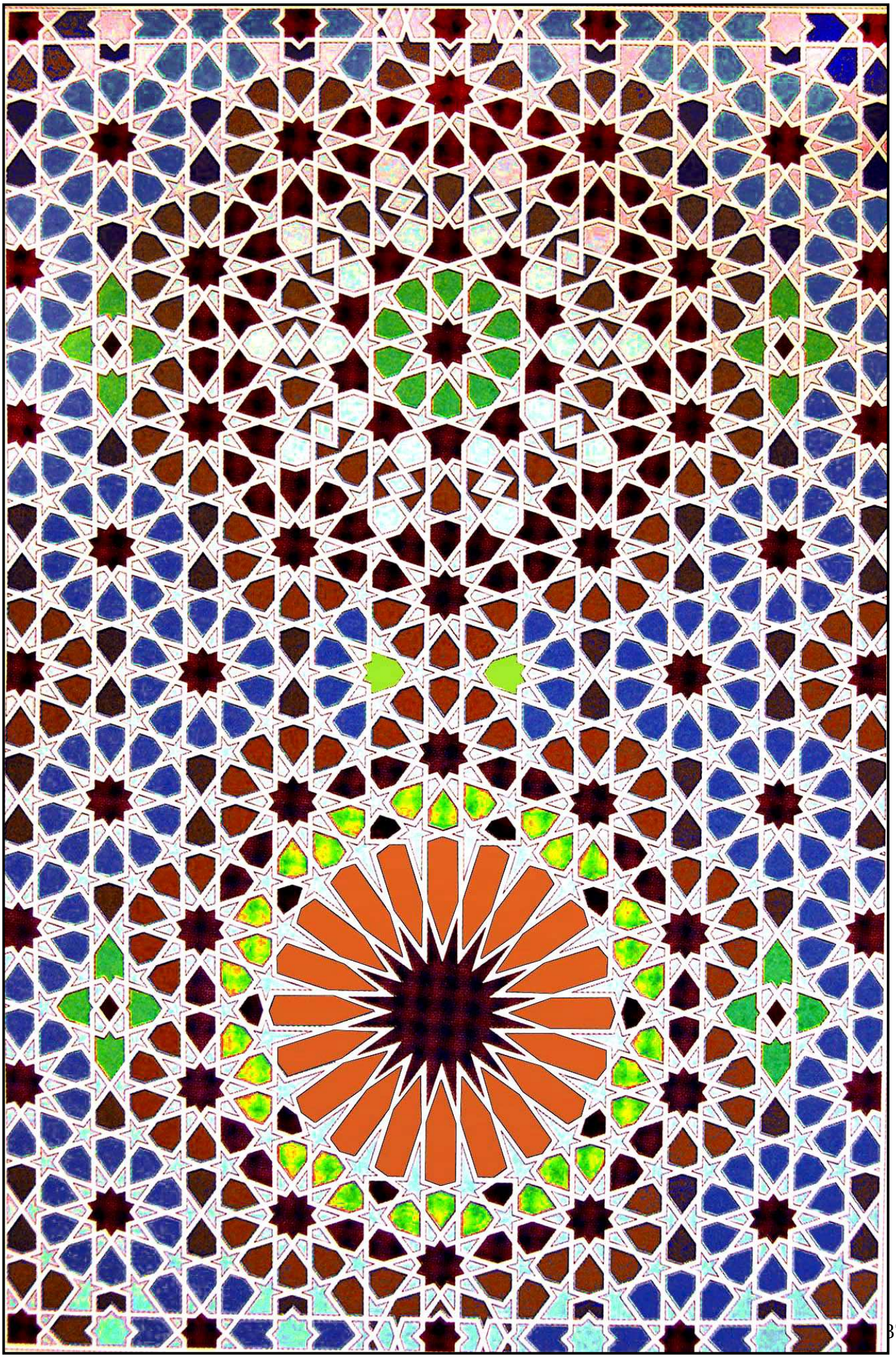
Donc

$$\frac{m'}{e'} = \frac{m}{e} = \frac{4}{\sqrt{5}-1}\left(\frac{8}{5+\sqrt{5}} + \frac{3\sqrt{5}-5}{5}\right)$$

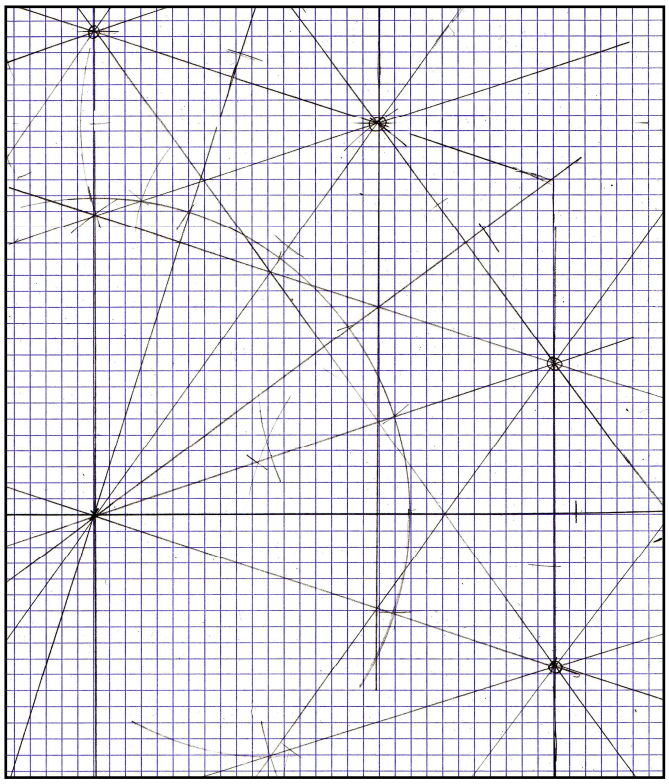
$$\frac{m}{e} = \frac{6\sqrt{5}+10}{5}$$

$$e = m \times 0{,}2135$$

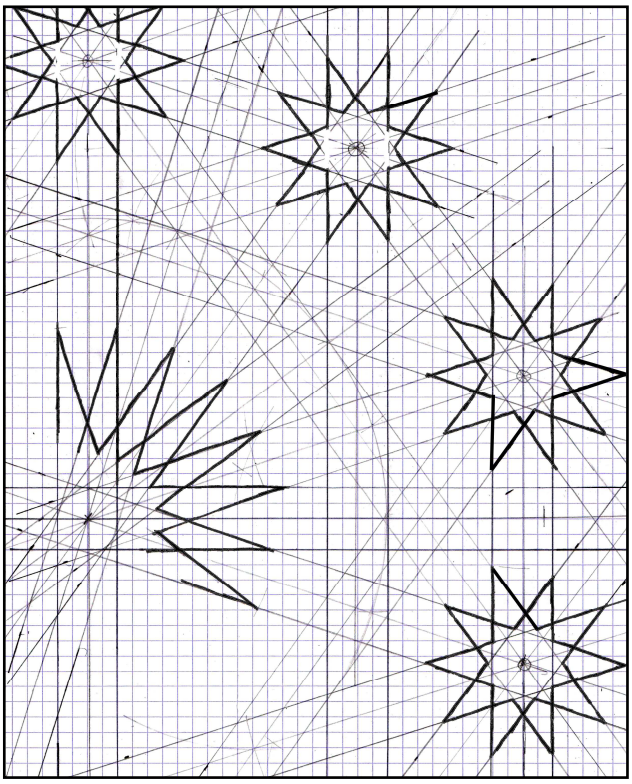




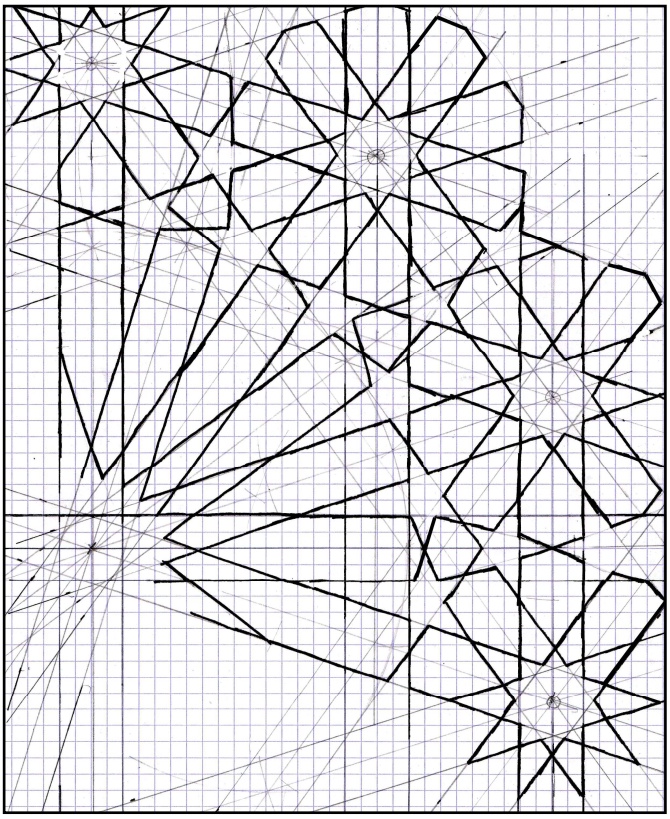
- *Construction :*



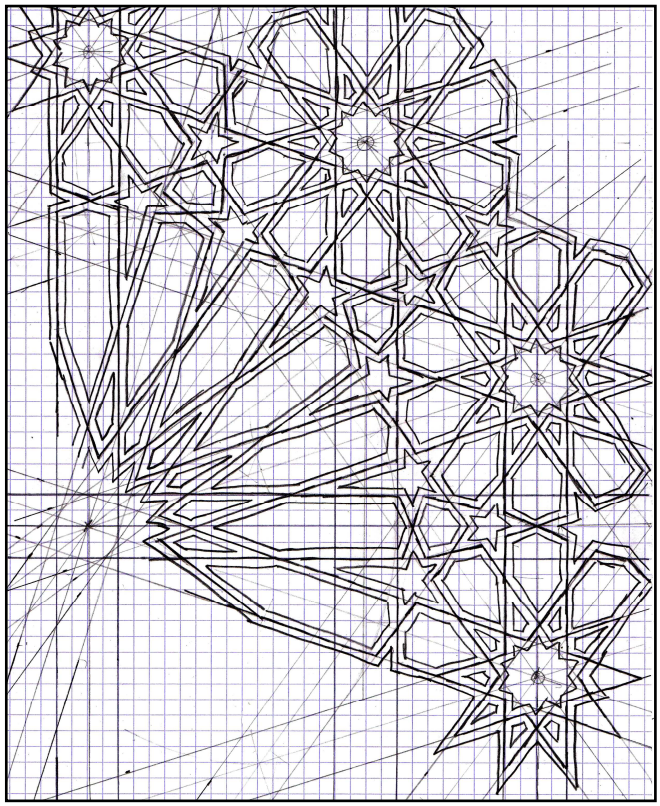
Placement du centre des étoiles m = 9,36



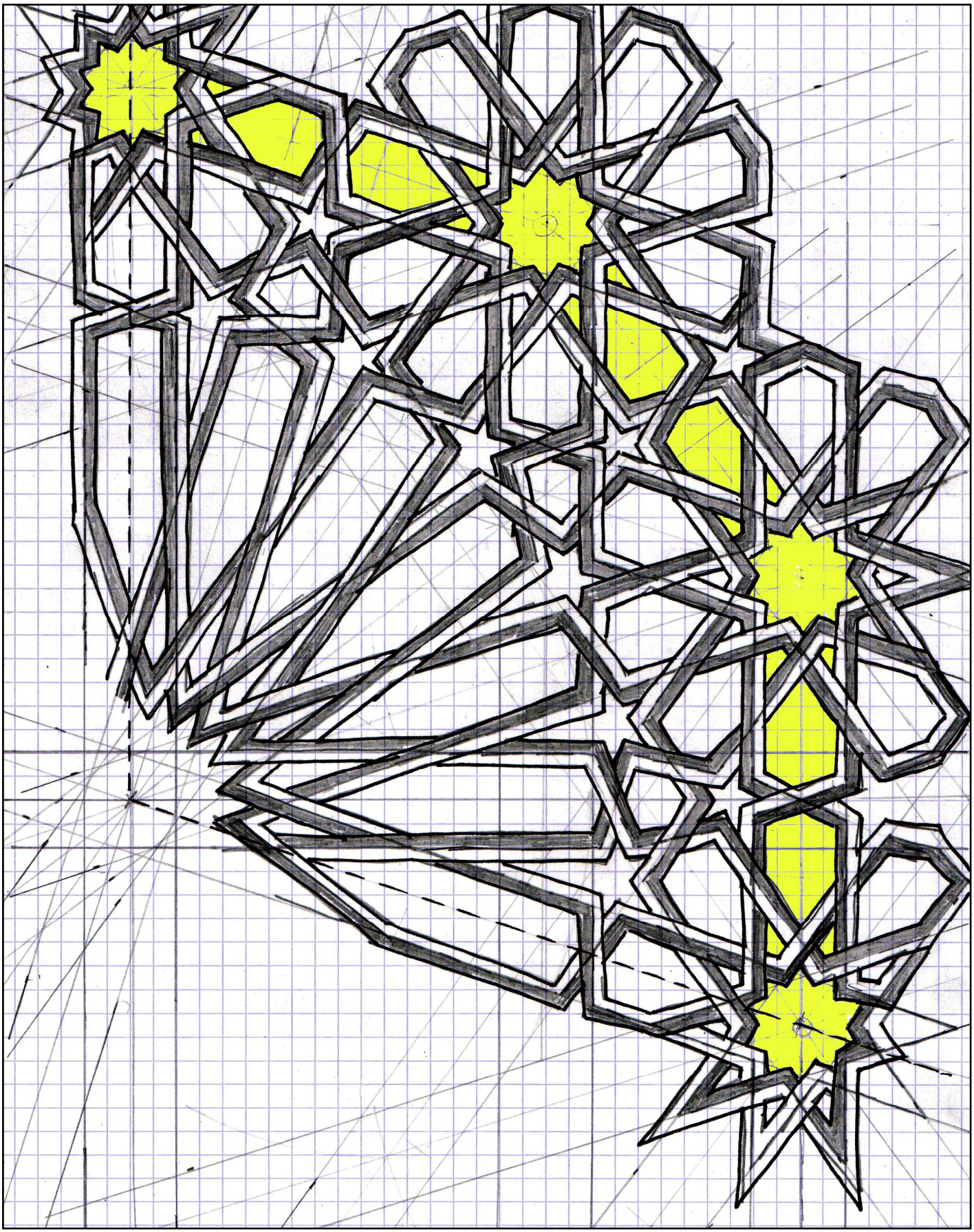
Bandage des axes des étoiles e = 2



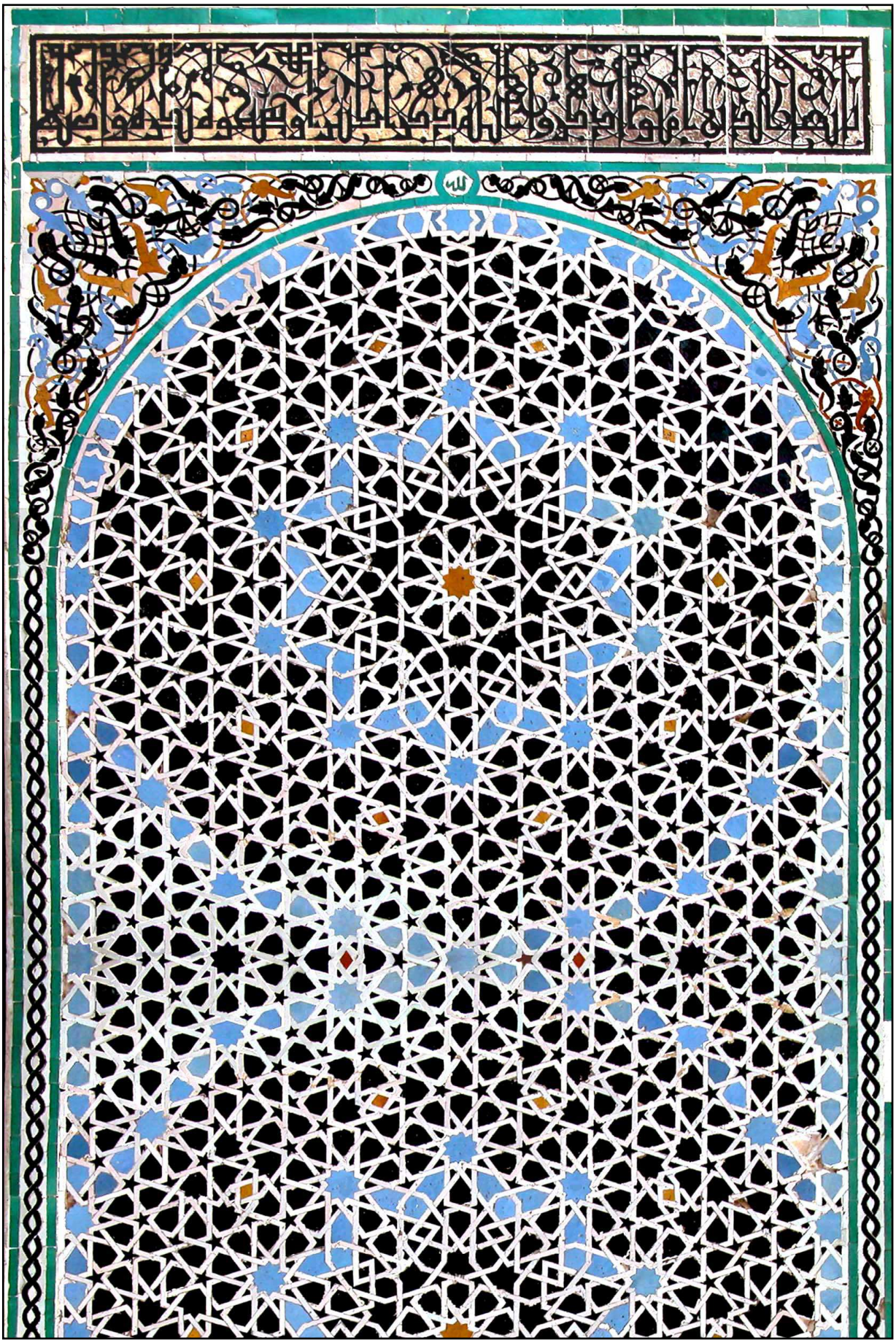
Fin du dessin.



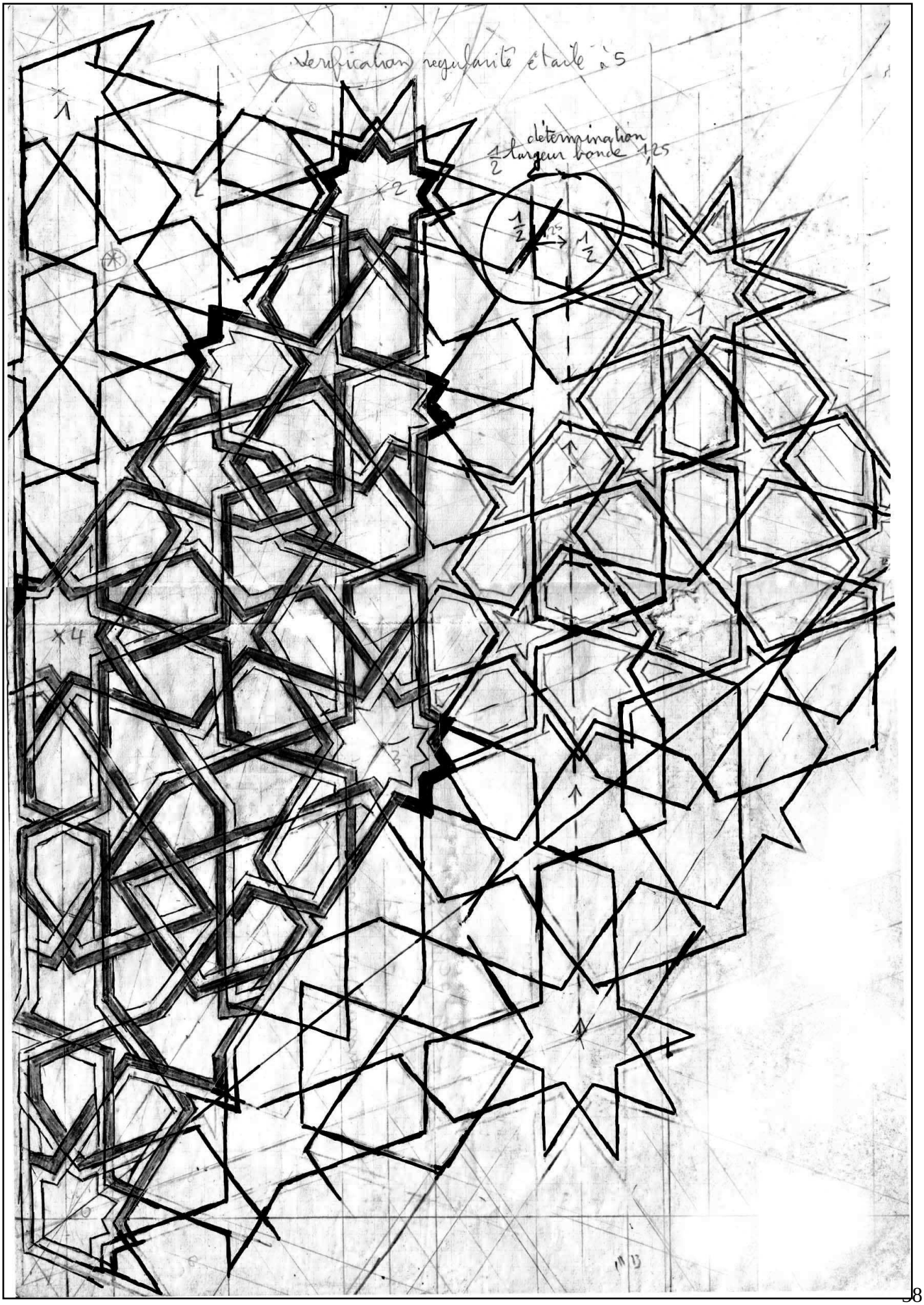
Construction des entrelacs type Alhambra: E = 0,4

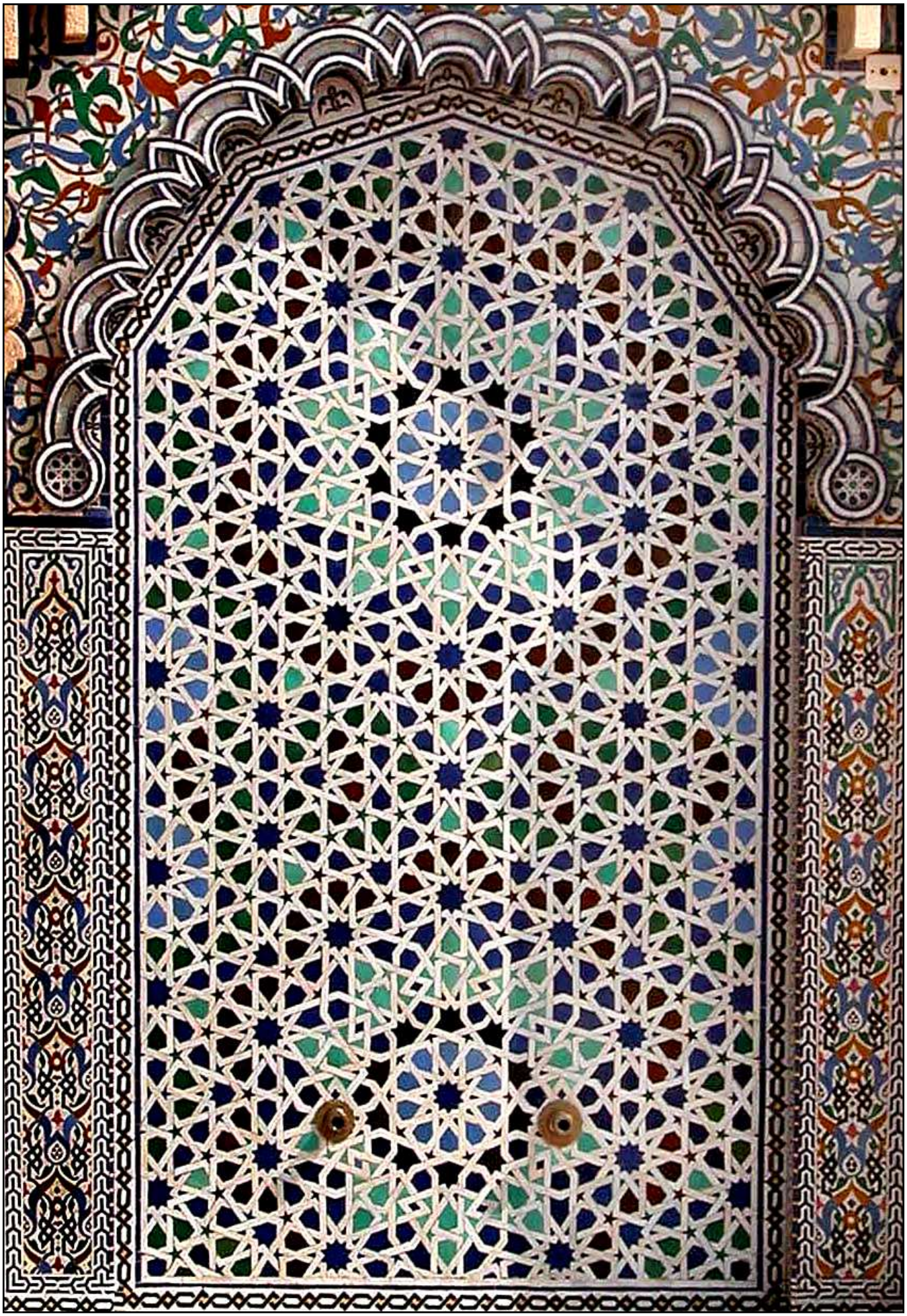


- *EXEMPLE : étude et construction d'un système:*

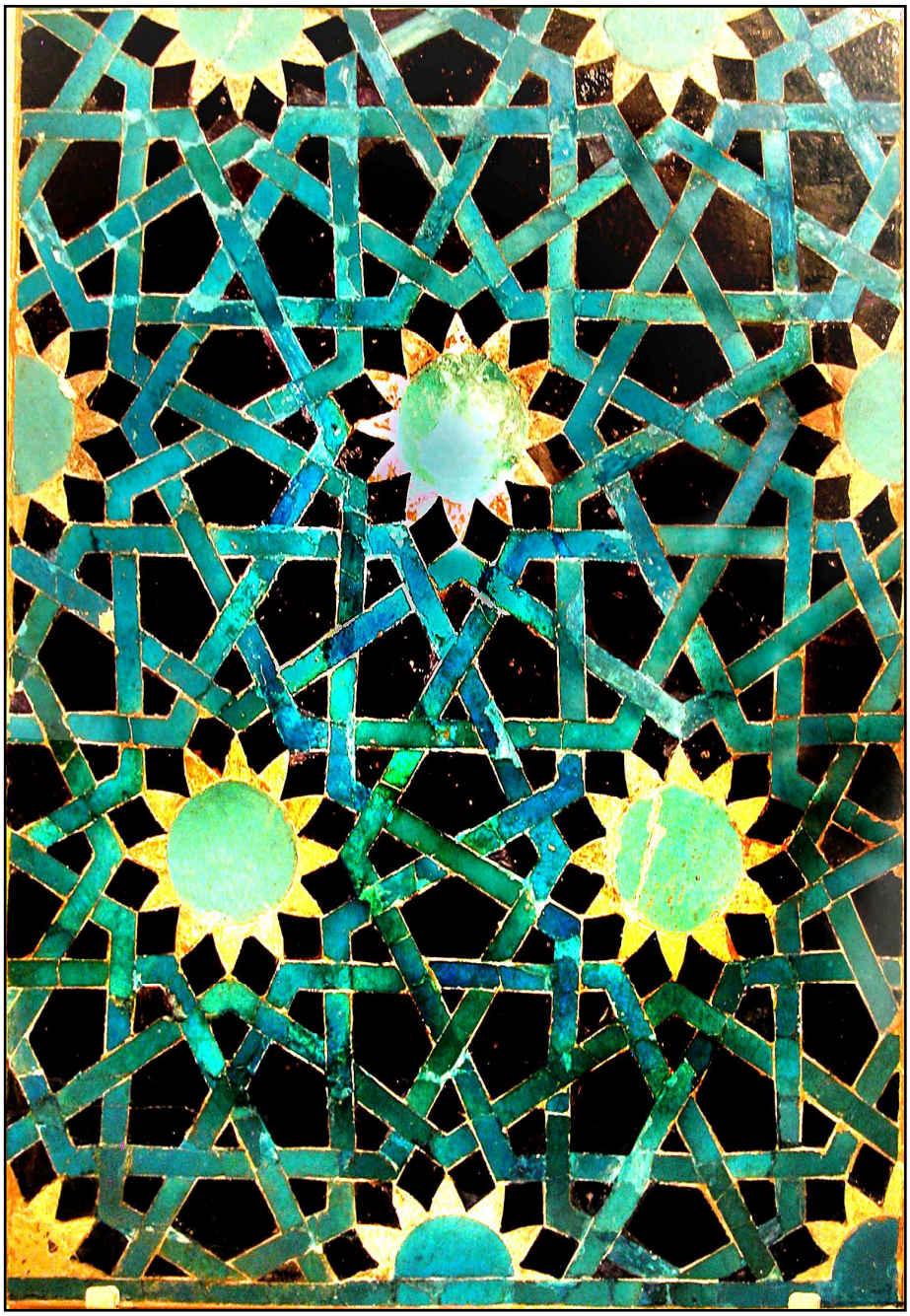
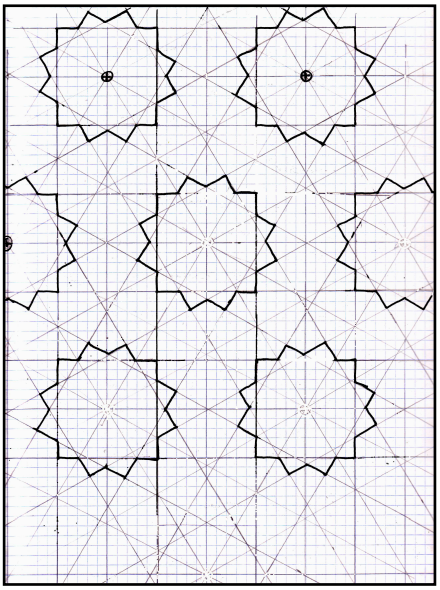
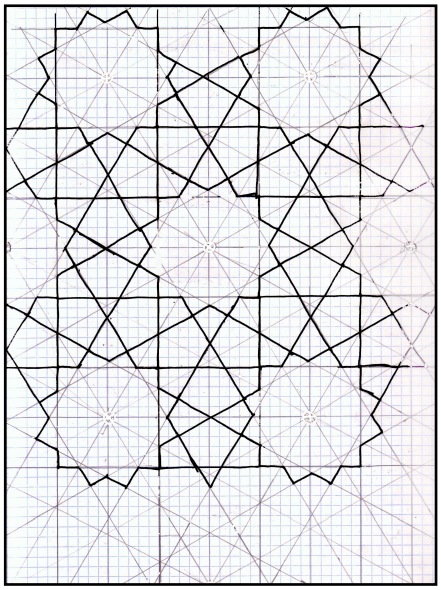
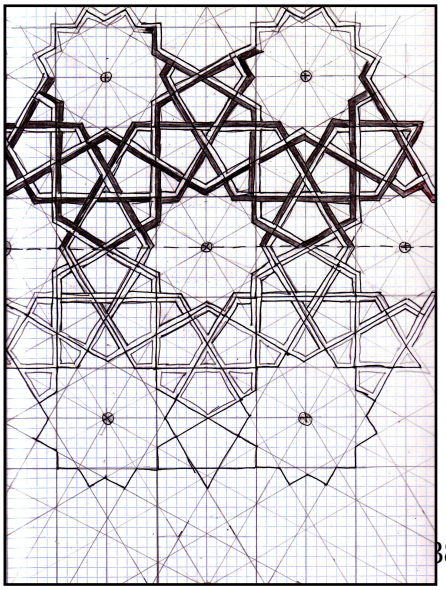


Médersa Attarine à Fès.


vérification régularité étoile à 5
détermination
1/2 largeur bande 1,25
1/2
1,25
1/2
1
2
1
4
3
1



Fontaine du mausolée Mohamed V à Rabat.



Panneau exposé au musée des arts Turcs à l'hippodrome d'Istanbul.